

# Ordered statistics decoding for semi-orthogonal linear block codes over random non-Gaussian channels

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**Abstract.** In this paper we propose suboptimal decoding of a low-rate linear code over non-gaussian channel within the ‘near-far’ problem OFDMA communication system. Proposed algorithm uses order statistics to mark out user signal in high interference conditions.

## 1 OFDMA system model

We consider OFDMA system where common channel space is distributed between independent users by a pseudorandom rule [4]. Each active user has for a time a ‘user block’ of the dimension  $q \times n$  (frequency–time) random positions. Each block is filled with messages by using  $q$ -ary multitone frequency modulation and  $q$ -ary codewords of a linear  $(n, 2, d)$ -code. Blocks from different users can interfere in random points by definition. Therefore, collision in a given point takes place with some probability that is a measure of traffic (or number of active users). Each codeword is mapped on a user block by the unique mask. Number of different masks is equal to the number of codewords. The decoding problem is equivalent to detection of the most likely mask for the received user block.

The common channel is a mobile channel with delay diversity and frequency spread. OFDM signal format includes cyclic prefix that transforms delay diversity into frequency transition coefficients  $h_i, i = 0, 1, \dots, T - 1$ , after Discrete Fourier Transform of order  $T$ . The probability density function of  $|h_i|^2$  is exponential  $\lambda e^{-\lambda|h_i|^2}$ .

In this paper we use COST 207 model for channel simulation.

## 2 COST 207 fading channel model

The COST 207 model was presented as an outdoor wireless channel model in [1]. This model specifies power gains, time delays, and Doppler spread for

four typical environments. These parameters were evaluated by numerous measurements performed in many countries, including the United Kingdom, France, and Sweden. The four typical environments were rural area (RA), typical urban area (TU), bad urban area (BU), and hilly terrain (HT). COST 207 standard provided both the continuous time formula and discrete taps model.

COST 207 model is often used as a multipath fading in reality. In this paper we use COST 207 HT to obtain multipath power gains and time delays for channel simulations.

### 3 Encoding

The general system has to use a concatenated encoding scheme. On the inner encoding level, we use a linear  $(n, 2, d)$ -code over  $\text{GF}(q)$  which is given as a concatenation of 2-extended Reed-Solomon MDR  $(q + 1, 2, q)$ -code and repetition code  $(\ell, 1, \ell)$ . So, the product code has the following parameters:  $n = (q + 1)\ell$ ,  $k = 2$ ,  $d = q\ell$ . In this paper we investigate the case  $q = 2^3$ ,  $\ell = 2$ . For this case, the code is equidistant. This means that for any couple of codewords it holds true that corresponding masks have exactly two common points.

The generating matrix of the  $(18, 2, 16)$ -code is

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & 1 & 0 & 1 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & 0 & 1 \end{pmatrix}$$

where  $\alpha$  is a prime element of the field  $\text{GF}(q)$ .

On the outer encoding level there should be used a linear code over  $\text{GF}q^2$  for example. The outer encoding level is not defined here. We investigate only the most critical part – inner coded modulation level.

### 4 Decoding

The optimal decoding in a case of Gaussian noise is achieved by selecting the mask with maximum received energy. Due to delay diversity and interfering signals from near users the noise statistics is going to be non-gaussian (or in the best, a mixture of different gaussian processes) and therefore the max energy decoder cannot be optimal. Moreover, we are not able to define the interfering point and even expected noise or signal power. Giving a non-gaussian channel, we need another metric that is more robust or tolerant to unknown noise statistics. One of known recipe is using order statistics. Order statistic was used firstly in Chase decoding and Forney GMD (generalized minimum distance) decoding [3]. In our case, only one of  $q^2$  masks is related to transmitted codeword and contains the signal samples. Thus, the presence of the signal samples in a mask acts as separating factor. One of possible solutions of detecting its presence is rank calculation of ordered series of measurements.

The rank calculation is known instrument in classic statistic theory. Using it in the proposed (RANK) decoding gives us an expected robustness. Another decoding algorithm using ordered statistics is the Order Statistics Normalized Envelope Detection Based Diversity Combining (OSN) [2]. Further we describe and compare both of them.

## 5 RANK decoder

Let  $\mathbf{X}$  be  $q \times n$  matrix of measurements of a received user block  $x_{i,j} = \|s_{i,j}h_{i,j} + \xi_{i,j}\|^2$ . Corresponding rank matrix  $\mathbf{R} = |r_{i,j}|_{q \times n}$  is defined by assigning to each energy measurement a number of measurements it exceeds:  $r_{i,j} = \#\{x_{l,m} < x_{i,j}\}$ ,  $x_{l,m}, x_{i,j} \in \mathbf{X}$ . Here  $\#$  means a number of elements in a set. Further decoding is performed on  $\mathbf{R}$ . For each codeword  $c_k$  corresponding mask matrix  $\mathbf{M}_k$  is applied to  $\mathbf{R}$ :  $f(\mathbf{R}, \mathbf{M}_k)$ . Mask matrix  $\mathbf{M}_k$  is a binary  $q \times n$  matrix with each column weight being exactly 1. Application function  $f()$  is element by element product. The result of the decoding is a codeword with maximum summary rank among different masks

$$D_{RANK} = \operatorname{argmax}_{k=1..q^2} \sum f(\mathbf{R}, \mathbf{M}_k).$$

## 6 Order statistics normalized envelope detection based diversity combining decoder

OSN decoder operates on order statistics of separate measurements corresponding to different masks. Let a vector of measurements  $x_k$  corresponding to codeword  $c_k$ :  $x_k = f(\mathbf{X}, \mathbf{M}_k)$  be reordered such way, that  $x'_{k1} \leq x'_{k2} \leq \dots \leq x'_{kn}$ . Then, decoding rule is

$$D_{OSN} = \operatorname{argmax}_{k=1..q^2} \sum_{t=1}^n \frac{x'_{kt}}{\chi_t},$$

where  $\chi_t = \sum_{i=1}^{q^2} x'_{it}$ .

## 7 Simulation results

Simulation of inner code decoding with OSN and RANK decoders were performed for different conditions. Power of interfering tones was set 20 and 30db above received signal. In both cases, collisions between users were simulated at 15, 25 and 30 percent of user area. Results of former case are given on figures 1 and 2. Results of latter case are given on figures 3–5. It may be seen that RANK decoder performs close to OSN when interfering tones power exceeds user signal power on 20 db and outperforms OSN in case of 30db.

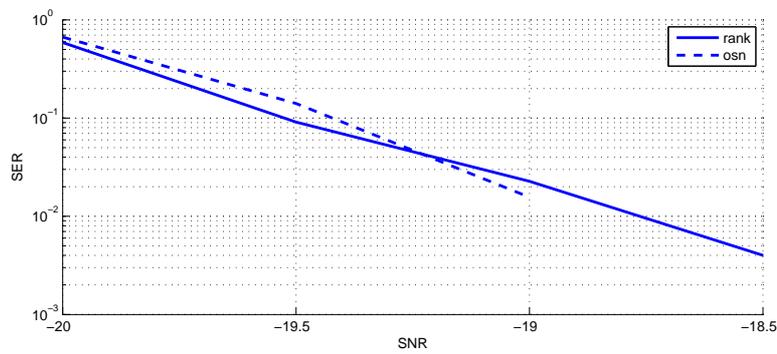


Figure 1: Collisions in 15% of user area with interference power 20db

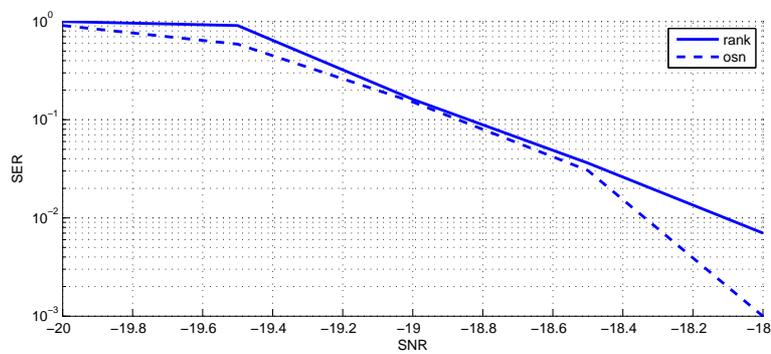


Figure 2: Collisions in 25% of user area with interference power 20db

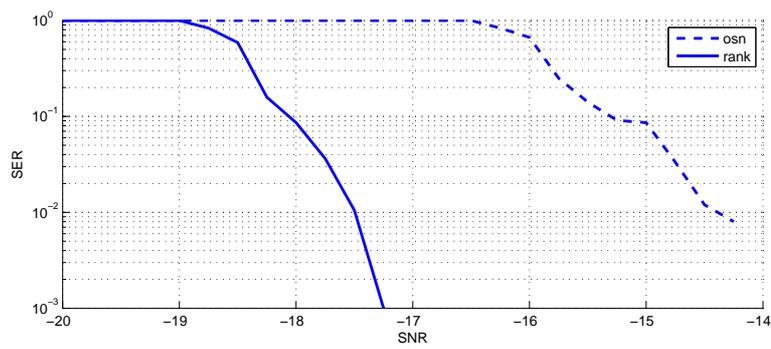


Figure 3: Collisions in 15% of user area with interference power 30db

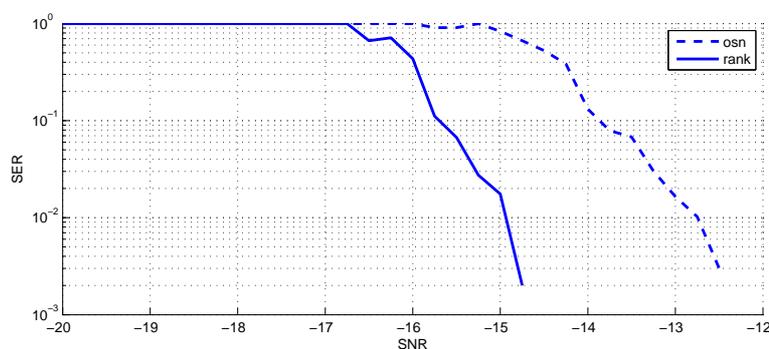


Figure 4: Collisions in 25% of user area with interference power 30db

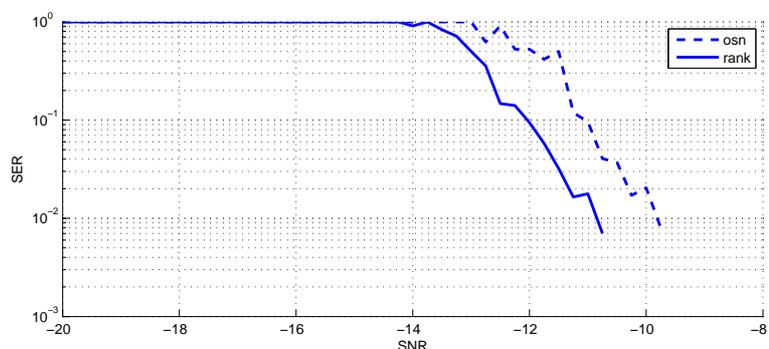


Figure 5: Collisions in 30% of user area with interference power 30db

## References

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