On the Binary Projective Codes with Dimension 6

Iliya Bouyukliev

***Abstract*— All binary projective codes of dimension up to 6 are classified. Information about the number of the codes with different minimum distances and different orders of automorphism groups is given.**

***Index Terms*— projective codes, code equivalence, canonical labelling, automorphism group, self-orthogonal codes.**

I. INTRODUCTION

Let F_2^n be the n-dimensional vector space over the Galois field F_2. The Hamming distance between two vectors of F_2^n is defined to be the number of coordinates in which they differ. A binary linear $[n, k, d]$-code is a k-dimensional linear subspace of F_2^n with a minimum distance d.

A central problem in coding theory is that of optimizing one of the parameters n, k and d for given values of the other two. Two versions are:

Problem 1: Find $d(n, k)$, the largest value of d for which a binary $[n, k, d]$-code exists.

Problem 2: Find $n(k, d)$, the largest value of k for which a binary $[n, k, d]$-code exists.

An $[n(k, d), k, d]$ code is called n-optimal and an $[n, k, d(n, k)]$ code is called d - optimal.

Another important problem is

Problem 3: Characterize all optimal binary codes.

The exact values of $n(k, d)$ were found for all d and $k \leq 8$(see [1],[15] [3] for dimensions 6,7 and 8). The exact values of $n(k, d)$ are also known for $d \leq 4$ and every k. Classification results on optimal binary codes can be found in [5], [14], [11], [12], [13], [16], [7], [4] etc. Classification results on projective codes are given in [10],[6].

An [n,k,d] code is called a projective code if its dual distance is at least 3. The binary projective codes do not have zero and repeated coordinates. In this paper we classify all binary projective codes of dimension up to six. For fixed n and k, most of the codes or their dual codes are optimal. Practically, we construct all d-optimal projective codes with dimension up to 6 and all n- optimal codes with $d = 3$ and $k < 58$.

In the second section we give some preliminary results. We describe the construction method in the third section. In the last section we give information for the number of the codes with different minimum distances and different orders of automorphism groups. We also list information on the number of self-orthogonal codes and codes with dual distance ≥ 4. In the end we present the parameters of the projective $[n, k \leq 6]$ codes with transitive automorphism groups.

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II. PRELIMINARIES

A generator matrix of a linear $[n, k]$-code C is any matrix of rank k with rows from C.

The weight $w(x)$ of a codeword x is defined to be the number of its non-zero entries.

For a given $[n, k, d]$ code $\mathcal{C}$, we denote by A_i the number of codewords of weight i in $\mathcal{C}$. The ordered $(n+1)$-tuple of integers $(A_0, A_1, \ldots, A_n)$ is called weight distribution of $\mathcal{C}$.

For every $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$ from F_2^n, $u \cdot v = u_1v_1 + u_2v_2 + \cdots + u_nv_n$ defines the inner product in F_2^n. The dual code of C is $C^\perp = \{v \in F_2^n : u \cdot v = 0 \forall u \in C\}$. If $C \subset C^\perp$, C is called self-orthogonal.

Two binary codes are equivalent if one can be obtained from the other by a permutation of coordinates. Thus two $[n, k]$ codes $\mathcal{C}_1$ and $\mathcal{C}_2$ are equivalent if there exists a monomial matrix $\mathbf{M}$ such that $(\mathbf{cM}) \in \mathcal{C}_2$ for every $\mathbf{c} \in \mathcal{C}_1$.

The permutation $\sigma \in S_n$ is an automorphism of C if $C = \sigma(C)$. The set of all automorphisms of C forms the automorphism group $Aut(C)$ of C. Two coordinates i and j are in the same orbit if there is an automorphism $\sigma \in Aut(C)$ such that $\sigma(i) = j$. If all coordinates of C are in the same orbit we call its group $Aut(C)$ transitive.

Let $C_1, C_2, \ldots, C_s$ are all different $[n, k]$ codes which are equivalent to the code B. We call the code B a canonical representative of its class of equivalence. Let $\sigma_i \in S_n$ be a permutation of the coordinates of the code C_i such that $\sigma_i(C_i) = B$, $i = 1, \ldots, s$. We call the permutation σ_i a canonical labelling map for the code C_i defined by B. As $\tau(C_i) = B \ \forall \tau \in \sigma_i Aut(C_i)$ the map σ_i is not unique except when $Aut(B) = \{id\}$. A canonical labelling of the coordinates of the code C_i is $(\sigma_i(1), \sigma_i(2), \ldots, \sigma_i(n))$. We call the l-th coordinate of the code C_i *last (or special)* in the canonical labelling if $\sigma_i(l) = n$ or l is in the same orbit with $\sigma^{-1}(n)$.

Remark 1. *How to choose the code B? We will present an example. Let c' and c'' be different codewords in a code C. We order the codewords of C in the following way: $c' \ll c''$ iff $c'_1 = c''_1, \ldots, c'_{i-1} = c''_{i-1}, c'_i < c''_i$. Denote the code C with ordered codewords by C^S ($c_i << c_j \iff i < j$, for every $c_i, c_j \in C^S$). Now we can compere codes D and E with the same dimension and length : $D << E \iff D^S \ll E^S$ ($D^S \ll E^S \iff d_1 = e_1, \ldots d_{(i-1)} = e_{(i-1)}, d_i \ll e_i$). For the canonical representative of the class of equivalence of the code C we can take the code B such that $B << \sigma(C)$ for every permutation $\sigma \in S_n$.*

Let $N = \{1, 2, \ldots, n\}$ be the set of the coordinates of the code C. An invariant of a coordinate of C is a function $f : N \to Z$ such that if i and j are in the same orbit with respect to $Aut(C)$ then $f(i) = f(j)$.

We call a partition π of N a set $\{N_1, N_2, \ldots, N_k\}$ of subsets $N_i \subset N$ such that $N_i \cap N_j = \emptyset$ for $i \neq j$ and $N = N_1 \cup N_2 \cup \cdots \cup N_k$.

The code C and f define the partition $\pi = \{N_1, N_2, \ldots, N_l\}$ such that two coordinates g_i, g_j are in the same subset of $N \iff f(g_i) = f(g_j)$. We denote $N_{i_1} \ll N_{i_2} \iff f(g_{i_1}) < f(g_{i_2})$, for $g_{i_1} \in N_{i_1}$ and $g_{i_2} \in N_{i_2}$. If $N_i \ll N_j \forall j \neq i, j = 1, \ldots, l$ we call the subset N_i minimal and denote it by $min(N)$.

Let us consider a canonical labelling map with a canonical representative B. Using the code B and the invariant f we can consider a new canonical labelling map defined by the code $B' = \tau(B)$ where τ is the permutation which maps $(1, 2, \ldots, n)$ to $(1, \ldots, a_1 - 1, a_1 + 1, \ldots, a_m - 1, a_m + 1, \ldots, n, a_1, a_2, \ldots, a_m)$, where $min(N) = \{a_1, \ldots, a_m\}$.

Remark 2. *More information about canonical labelling map can be found in the remarkable work of Brendan McKay [8].*

III. Codes construction

In our work we have used a McKay approach [9] for isomorphism rejection.

Let k be a positive integer and H be a $k \times (2^k - 1)$ matrix whose columns are the distinct non-zero vectors of F_2^k. Then the code having H as a generator matrix is called a simplex code. Let us denote by M the $(2^k - 1) \times (2^k - 1)$ matrix of the nonzero codewords of the simplex code with dimension k . To construct every projective code we use the fact that it is a punctured version of the corresponding simplex code. In other words, the codewords of every projective code C are obtained by taking some fixed number of the columns of the matrix M. We will say that the code C is defined by these columns of M.

The main idea using McKay-type approach is to construct recursively new child codes from parent codes. In our case if a parent code is defined by n columns of the matrix M the child code will be defined by these columns plus a new column from M. As child codes will be accepted only those codes that pass a parent test and an isomorphism test.

In the parent test we need a canonical labelling [9] of the coordinates of the codes. The parent test can be passed by those child codes which last added coordinate is *last* in the canonical labelling. From the child codes which passed the parent test we take only one representative from each class of equivalence.

The construction algorithm. Start from an empty set and recursively do the following. For a given code in the search tree, construct all possible child codes obtained by adding one coordinate. For each such child, carry out the parent test and, for those who survive the parent test, carry out isomorph rejection with the isomorphism test among those codes that come from the same parent.

An advantage of the algorithm for our investigation is that we have to find an isomorphism only between the children of one parent.

We will prove that our algorithm generates all nonequivalent binary projective codes of a given length n by induction to the depth of the recursion.

Lemma 1: The given algorithm generates all nonequivalent binary projective codes of a given length $n \geq 3$.

Proof: We start from an empty set. As projective codes of lengths 1 and 2 do not exist, we take the [1,1] code in depth 1, and the [2,2] code in depth 2. In depth 3 we obtain two projective codes of length 3 as children of the [2,2] code with parameters [3,3,1] and [3,2,2] and they are all projective codes of this length.

Suppose that we have constructed all nonequivalent projective codes of length $n - 1 > 2$ in depth $n - 1$ and they are $C_1, C_2, \ldots, C_t$. We have to prove that:

1) any binary projective code of length n is equivalent to a code obtained by our algorithm;

2) The child codes of one parent are not equivalent to the child codes of another parent.

1) Let C be a binary projective code of length n. If C^* is the code obtained from C by deleting the last coordinate then C^* is a projective code of length $n-1$. Therefore C^* is equivalent to one of the codes $C_1, \ldots, C_t$. Without loss of generality $C^* \cong C_1$. Let A^* and A_1 are $(2^k - 1) \times (n - 1)$ matrices whose rows are the all nonzero codewords of C^* and C_1, respectively. Then $G_1 = MG^*M^*$ where M is a monomial $(2^k - 1) \times (2^k - 1)$ and M^* is a monomial $(n - 1) \times (n - 1)$ matrix. It follows that the code C' generated by the matrix $(G_1|Mx)$, where x is the column corresponding to the last coordinate of C, is equivalent to C. Let $(a_1, a_2, \ldots, a_n)$ be the canonical labelling of the coordinates of C'. If n is *last* in the canonical labelling then we construct C' as a child code from C_1. Otherwise, if l is *last* in the canonical labelling we can change the l-th and the n-th coordinates. In the new code C'' n will be *last* in the canonical labelling of the coordinates. The first $n - 1$ coordinates of C'' define a code equivalent to one of the codes $C_1, \ldots, C_t$, say C_2. Similarly, we can take a vector-column x_2 such that $(C_2|x_2) \cong C''$. Since the first $n-1$ coordinates of the codes $(C_2|x_2)$, C'' and their canonical representative B define equivalent codes, and n-th coordinate is *last* for the C'' and B, then it is *last* for the code $(C_2|x_2)$, too.

2) Suppose that $(C|x) \cong (C^*|x^*)$ where $(C|x)$ and $(C^*|x^*)$ are projective codes of length n which have passed the parent test. Hence the n-th coordinate is *last* for both codes. Therefore $C \cong C^* \cong B^*$ where B^* is the canonical representative of $(C|x)$ and $(C^*|x^*)$ without the n-th coordinate. ■

For an isomorphism test we use the algorithm from [2]. This algorithm gives us also the canonical labelling map. For a parent test we use new canonical labelling map defined by this one and an invariant f which we will describe now. Consider an $[n, k, d]$ code C and order its nonzero codewords. To C we juxtapose a vector t of length $2^k - 1$ such that $t_i = wt(v_i)$ where v_i is the i-th codeword. We define $f(i)$ in the following way:

$$f(i) = \sum_{j \in T_i} t_j(t_j + 1) + \sum_{j \in T \setminus T_i} (t_j - 1)^3$$

where $T_i = \{j : v_j[i] = 0\}$ and $T = \{1, 2, \ldots, 2^k - 1\}$. This invariant collects information about the weight enumerator of the code obtained from C by deleting the i-th coordinate and also how we reduce the spectrum of C to obtain the new spectrum. The complexity of the calculating of the vector t of a child code from the corresponding vector of its parent code is $O(2^k)$. The complexity of the calculating of f for all coordinates is $O(n2^k)$.

For the parent test we calculate the invariant f first. If $n \notin min(N)$ we can reject the code because the n-th coordinate is not *last* in the canonical labelling map defined by using f. So we need the concrete canonical labelling only if $n \in min(N)$. The canonical labelling is more computationally expensive than the calculating of the invariant f in the cases of codes with specific structure or reach automorphism group.

Remark 3. *In geometrical aspect, we can define a binary projective $[n, k, d]$ code as a set $\mathcal{C}$ of n points in $PG(k-1, 2)$ such that (a) each hyperplane of $PG(k-1, 2)$ meets $\mathcal{C}$ in at most $n-d$ points and (b) there is a hyperplane meeting $\mathcal{C}$ in exactly $n-d$ points. This definition is equivalent to the one given in section 2. It is easy to see that projective codes are equivalent if and only if the corresponding sets of points are projectively equivalent. In this work all nonequivalent sets of points in $PG(5, 2)$ (all orbits under $PGL(6, 2)$ of projective n-subsets of $PG(5, 2)$) have been computed.*

IV. Classification results

We run the algorithm to depth 32. The calculation took about four days of CPU time on a 1800 MHz PC. All other nonequivalent codes with dimension 6 and length up to 63 we can find as a complement of these codes to the simplex code. Practically we compute codes with length 32 twice for verification: first with the algorithm and then as a complement of codes with length 31. We find 284625281 nonequivalent projective codes with length up to 32. The number of all codes which we consider (candidates for children codes) is 8252302118. For these codes we calculate only invariant f. The number of the codes for which we compute canonical labelling is 331742121. We save information for all this codes - generator matrices in compressive form, minimum weight, maximum weight, order of automorphism group (not more than a hundred symbols for a code) in a 9 GB file. Generator matrix we write as a hexadecimal vector. For example binary vector with length 64
$v = (1101, 0010, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000, 0000)$ has the following hexadecimal presentation: $b400000000000000$. We can consider the binary presentation of a nonzero coordinate of a vector v as a column vector of a generator matrix of the code C. The nonzero coordinates of the vector v are $\{1, 2, 4, 7\}$ and the corresponding generator matrix is:

$$G = \begin{pmatrix} 1001 \\ 0101 \\ 0011 \end{pmatrix}$$

In Appendix 1 we present general information on the binary projective codes of dimensions 4, 5 and 6. For each length the common number of the nonequivalent codes is given. Then the number of codes having the corresponding minimum distance is written as a power of this minimum distance. After the string 'AUT' there is information for the order of the automorphism group of the codes with given length and dimension. The number of the codes having automorphism groups with the same order is written first and the order of the automorphism groups of the codes is written as a power.

Similar information (without information on automorphism groups) about binary projective self-orthogonal codes and binary projective codes with dual distance 4 with dimensions 5 and 6 we give in Appendix 2 and Appendix 3.

The generator matrices, weight spectrums, and automorphism groups order on all binary projective codes with transitive automorphism group are listed in Appendix 4.

The complete list with all determined properties of the codes can be found at
http://www.moi.math.bas.bg/∼iliya.

V. Appendix

Appendix 1 Number of the projective codes in dimension 4, 5 and 6.

$\mathbf{k = 4\ n = 4}$ #1 1^{1},
AUT: 1^{24},
$\mathbf{k = 4\ n = 5}$ #3 1^{2}, 2^{1},
AUT: 1^{12}, 1^{120}, 1^{24},
$\mathbf{k = 4\ n = 6}$ #4 1^{1}, 2^{3},
AUT: 1^{72}, 1^{8}, 1^{12}, 1^{48},
$\mathbf{k = 4\ n = 7}$ #5 1^{1}, 2^{3}, 3^{1},
AUT: 1^{48}, 2^{8}, 1^{168}, 1^{24},
$\mathbf{k = 4\ n = 8}$ #6 1^{1}, 2^{2}, 3^{2}, 4^{1},
AUT: 1^{24}, 1^{1344}, 2^{8}, 1^{48}, 1^{168},
$\mathbf{k = 4\ n = 9}$ #5 2^{1}, 3^{2}, 4^{2},
AUT: 1^{192}, 1^{8}, 1^{12}, 1^{72}, 1^{48},
$\mathbf{k = 4\ n = 10}$ #4 3^{1}, 4^{3},
AUT: 1^{64}, 1^{24}, 1^{12}, 1^{120},
$\mathbf{k = 4\ n = 11}$ #3 4^{2}, 5^{1},
AUT: 1^{192}, 1^{48}, 1^{24},
$\mathbf{k = 4\ n = 12}$ #2 5^{1}, 6^{1},
AUT: 1^{48}, 1^{576},
$\mathbf{k = 4\ n = 13}$ #1 6^{1},
AUT: 1^{192},
$\mathbf{k = 4\ n = 14}$ #1 7^{1},
AUT: 1^{1344},
$\mathbf{k = 4\ n = 15}$ #1 8^{1},
AUT: 1^{20160},
$\mathbf{k = 5\ n = 5}$ #1 1^{1},
AUT: 1^{120},
$\mathbf{k = 5\ n = 6}$ #4 1^{3}, 2^{1},
AUT: 1^{36}, 1^{720}, 1^{120}, 1^{48},
$\mathbf{k = 5\ n = 7}$ #8 1^{4}, 2^{4},
AUT: 2^{72}, 3^{48}, 1^{16}, 1^{12}, 1^{144},
$\mathbf{k = 5\ n = 8}$ #15 1^{5}, 2^{10},
AUT: 5^{48}, 1^{24}, 1^{12}, 3^{8}, 2^{16}, 1^{384}, 1^{32}, 1^{168},
$\mathbf{k = 5\ n = 9}$ #29 1^{6}, 2^{19}, 3^{4},
AUT: 2^{144}, 3^{4}, 1^{12}, 2^{24}, 6^{8}, 5^{48}, 2^{336}, 2^{16}, 2^{32}, 1^{72}, 1^{384}, 1^{192}, 1^{1344},
$\mathbf{k = 5\ n = 10}$ #46 1^{5}, 2^{24}, 3^{14}, 4^{3},
AUT: 3^{12}, 1^{10}, 1^{144}, 1^{1008}, 10^{8}, 1^{720}, 5^{4}, 2^{2}, 7^{48}, 4^{16}, 3^{32}, 3^{192}, 1^{1920}, 1^{72}, 1^{24}, 1^{336}, 1^{384},
$\mathbf{k = 5\ n = 11}$ #64 1^{4}, 2^{21}, 3^{27}, 4^{12},
AUT: 5^{48}, 11^{4}, 6^{2}, 9^{8}, 3^{12}, 9^{16}, 1^{10}, 5^{32}, 1^{1920}, 2^{96}, 4^{24}, 2^{192}, 2^{120}, 1^{6}, 1^{384}, 2^{64},
$\mathbf{k = 5\ n = 12}$ #89 1^{3}, 2^{16}, 3^{32}, 4^{38},
AUT: 12^{16}, 9^{12}, 12^{2}, 13^{4}, 2^{36}, 11^{8}, 2^{6}, 6^{48}, 2^{96}, 6^{32}, 5^{24}, 1^{1152}, 1^{1}, 2^{64}, 1^{2304}, 3^{192}, 1^{128},
$\mathbf{k = 5\ n = 13}$ #112 1^{2}, 2^{11}, 3^{25}, 4^{65}, 5^{9},
AUT: 21^{4}, 1^{36}, 11^{12}, 8^{48}, 4^{6}, 15^{2}, 16^{8}, 12^{16}, 7^{32}, 1^{1152}, 3^{1}, 1^{144}, 2^{64}, 3^{192}, 1^{24}, 1^{768}, 2^{576}, 2^{96}, 1^{128},
$\mathbf{k = 5\ n = 14}$ #128 1^{1}, 2^{6}, 3^{16}, 4^{65}, 5^{37}, 6^{3},
AUT: 10^{48}, 27^{4}, 9^{12}, 15^{16}, 16^{8}, 3^{6}, 24^{2}, 2^{24}, 3^{1}, 3^{64}, 6^{32}, 3^{192}, 1^{2688}, 2^{128}, 1^{768}, 2^{96}, 1^{384},
$\mathbf{k = 5\ n = 15}$ #144 1^{1}, 2^{3}, 3^{9}, 4^{46}, 5^{63}, 6^{21}, 7^{1},
AUT: 2^{720}, 13^{48}, 2^{24}, 16^{16}, 30^{2}, 2^{10}, 7^{6}, 26^{4}, 3^{1}, 18^{8}, 4^{12}, 3^{64}, 1^{2688},

$4^{192}, 1^{20160}, 4^{32}, 1^{144}, 2^{128}, 1^{2304}, 1^{576}, 1^{36}, 1^{384}, 1^{1344},$

k = 5 n = 16 #145 $1^{1}, 2^{2}, 3^{4}, 4^{25}, 5^{53}, 6^{57}, 7^{2}, 8^{1},$

AUT: $2^{720}, 1^{144}, 13^{48}, 2^{10}, 7^{6}, 26^{4}, 16^{16}, 18^{8}, 30^{2}, 3^{1}, 3^{64}, 1^{1344}, 1^{322560}, 4^{192}, 1^{384}, 4^{32}, 1^{36}, 4^{12}, 1^{576}, 2^{24}, 2^{128}, 1^{2304}, 1^{2688}, 1^{20160},$

k = 5 n = 17 #129 $2^{1}, 3^{2}, 4^{11}, 5^{29}, 6^{71}, 7^{14}, 8^{1},$

AUT: $15^{16}, 10^{48}, 24^{2}, 27^{4}, 16^{8}, 2^{24}, 9^{12}, 3^{1}, 3^{6}, 3^{64}, 6^{32}, 1^{21504}, 3^{192}, 2^{96}, 1^{384}, 2^{128}, 1^{768}, 1^{2688},$

k = 5 n = 18 #113 $3^{1}, 4^{5}, 5^{12}, 6^{49}, 7^{41}, 8^{5},$

AUT: $12^{16}, 16^{8}, 8^{48}, 21^{4}, 15^{2}, 4^{6}, 3^{1}, 11^{12}, 1^{36}, 7^{32}, 3^{192}, 1^{3072}, 2^{576}, 2^{96}, 1^{128}, 2^{64}, 1^{144}, 1^{1152}, 1^{768}, 1^{24},$

k = 5 n = 19 #91 $4^{2}, 5^{5}, 6^{21}, 7^{41}, 8^{22},$

AUT: $6^{48}, 12^{16}, 9^{12}, 13^{4}, 12^{2}, 2^{6}, 5^{24}, 3^{192}, 1^{9216}, 1^{768}, 11^{8}, 6^{32}, 1^{128}, 2^{64}, 2^{36}, 1^{1}, 1^{1152}, 2^{96}, 1^{2304},$

k = 5 n = 20 #67 $5^{2}, 6^{8}, 7^{20}, 8^{34}, 9^{3},$

AUT: $1^{10}, 11^{4}, 5^{48}, 2^{120}, 1^{768}, 1^{3072}, 2^{384}, 3^{12}, 2^{64}, 4^{24}, 9^{16}, 9^{8}, 2^{192}, 5^{32}, 6^{2}, 1^{6}, 2^{96}, 1^{1920},$

k = 5 n = 21 #50 $6^{3}, 7^{8}, 8^{24}, 9^{13}, 10^{2},$

AUT: $1^{720}, 4^{192}, 2^{1920}, 1^{1024}, 2^{384}, 1^{72}, 10^{8}, 3^{12}, 7^{48}, 1^{144}, 1^{1008}, 5^{4}, 4^{16}, 1^{336}, 3^{32}, 2^{2}, 1^{10}, 1^{24},$

k = 5 n = 22 #34 $7^{3}, 8^{11}, 9^{13}, 10^{7},$

AUT: $1^{1152}, 1^{128}, 2^{192}, 1^{768}, 1^{3072}, 5^{48}, 6^{8}, 2^{336}, 2^{24}, 1^{1344}, 2^{144}, 3^{4}, 2^{16}, 2^{32}, 1^{384}, 1^{12}, 1^{72},$

k = 5 n = 23 #21 $8^{4}, 9^{7}, 10^{9}, 11^{1},$

AUT: $1^{768}, 2^{128}, 1^{2688}, 2^{384}, 1^{21504}, 5^{48}, 3^{8}, 1^{168}, 1^{12}, 1^{24}, 2^{16}, 1^{32},$

k = 5 n = 24 #14 $9^{3}, 10^{7}, 11^{3}, 12^{1},$

AUT: $1^{384}, 1^{64512}, 2^{128}, 1^{768}, 1^{2688}, 1^{16}, 1^{12}, 2^{72}, 3^{48}, 1^{144},$

k = 5 n = 25 #9 $10^{3}, 11^{4}, 12^{2},$

AUT: $1^{9216}, 1^{128}, 1^{192}, 1^{1152}, 1^{768}, 1^{48}, 1^{36}, 1^{120}, 1^{720},$

k = 5 n = 26 #5 $11^{2}, 12^{3},$

AUT: $1^{3072}, 1^{384}, 1^{192}, 1^{1920}, 1^{120},$

k = 5 n = 27 #3 $12^{2}, 13^{1},$

AUT: $1^{9216}, 1^{2304}, 1^{384},$

k = 5 n = 28 #2 $13^{1}, 14^{1},$

AUT: $1^{2304}, 1^{64512},$

k = 5 n = 29 #1 $14^{1},$

AUT: $1^{21504},$

k = 5 n = 30 #1 $15^{1},$

AUT: $1^{322560},$

k = 5 n = 31 #1 $16^{1},$

AUT: $1^{9999360},$

k = 6 n = 6 #1 $1^{1},$

AUT: $1^{720},$

k = 6 n = 7 #5 $1^{4}, 2^{1},$

AUT: $2^{144}, 1^{240}, 1^{5040}, 1^{720},$

k = 6 n = 8 #14 $1^{8}, 2^{6},$

AUT: $1^{720}, 4^{144}, 3^{48}, 1^{240}, 1^{24}, 1^{1152}, 1^{192}, 1^{72}, 1^{96},$

k = 6 n = 9 #38 $1^{15}, 2^{23},$

AUT: $9^{48}, 1^{1296}, 2^{192}, 2^{96}, 3^{32}, 2^{12}, 2^{72}, 1^{128}, 3^{24}, 5^{16}, 3^{144}, 1^{8}, 2^{288}, 1^{384}, 1^{336},$

k = 6 n = 10 #105 $1^{29}, 2^{72}, 3^{4},$

AUT: $5^{32}, 19^{16}, 4^{96}, 2^{576}, 7^{12}, 12^{8}, 4^{64}, 1^{128}, 5^{24}, 5^{144}, 17^{48}, 3^{1008}, 8^{4}, 1^{120}, 2^{288}, 4^{192}, 1^{72}, 2^{384}, 1^{3840}, 1^{336}, 1^{2688},$

k = 6 n = 11 #273 $1^{46}, 2^{188}, 3^{37}, 4^{2},$

AUT: $9^{96}, 1^{576}, 12^{24}, 49^{16}, 18^{32}, 10^{192}, 1^{128}, 8^{12}, 1^{4032}, 44^{8}, 2^{1008}, 41^{4}, 17^{2}, 1^{6}, 1^{120}, 2^{10}, 8^{144}, 26^{48}, 2^{288}, 5^{64}, 1^{720}, 6^{384}, 1^{3840}, 2^{1920}, 1^{1344}, 2^{336}, 1^{8064}, 1^{1152},$

k = 6 n = 12 #700 $1^{64}, 2^{395}, 3^{214}, 4^{27},$

AUT: $42^{32}, 15^{64}, 85^{16}, 24^{96}, 14^{192}, 5^{144}, 2^{1344}, 5^{1152}, 1^{20}, 137^{4}, 88^{2}, 16^{12}, 24^{1}, 6^{6}, 144^{8}, 23^{24}, 36^{48}, 1^{36}, 3^{72}, 2^{288}, 1^{432}, 1^{576}, 2^{240}, 2^{120}, 2^{10}, 7^{384}, 4^{128}, 2^{1920}, 1^{23040}, 1^{3840}, 1^{672}, 1^{336}, 1^{1536}, 1^{256},$

k = 6 n = 13 #1794 $1^{89}, 2^{695}, 3^{757}, 4^{253},$

AUT: $34^{64}, 78^{32}, 165^{16}, 5^{1152}, 20^{192}, 2^{4032}, 47^{24}, 378^{4}, 360^{2}, 16^{6}, 180^{1}, 329^{8}, 36^{12}, 5^{144}, 54^{48}, 41^{96}, 3^{72}, 1^{720}, 4^{576}, 2^{20}, 6^{36}, 9^{128}, 8^{384}, 1^{23040}, 2^{768}, 1^{3840}, 1^{240}, 1^{672}, 3^{256}, 1^{1536}, 1^{2304},$

k = 6 n = 14 #4579 $1^{112}, 2^{1071}, 3^{1840}, 4^{1545}, 5^{11},$

AUT: $310^{16}, 1^{56448}, 62^{24}, 945^{4}, 1^{168}, 673^{8}, 1126^{2}, 928^{1}, 149^{32}, 73^{48}, 52^{96}, 6^{576}, 6^{1152}, 76^{12}, 24^{192}, 48^{64}, 36^{6}, 1^{14}, 2^{72}, 6^{36}, 23^{128}, 3^{768}, 9^{384}, 3^{1536}, 2^{2304}, 5^{144}, 3^{288}, 1^{4032}, 3^{256}, 1^{4608}, 1^{512},$

k = 6 n = 15 #11635 $1^{128}, 2^{1460}, 3^{3398}, 4^{6119}, 5^{525}, 6^{5},$

AUT: $240^{32}, 1^{360}, 1220^{8}, 1^{120}, 2101^{4}, 4^{1152}, 552^{16}, 58^{96}, 3756^{1}, 3094^{2}, 132^{12}, 93^{48}, 88^{6}, 66^{64}, 91^{24}, 2^{3}, 6^{72}, 1^{240}, 2^{720}, 3^{36}, 4^{144}, 22^{192}, 50^{128}, 17^{384}, 7^{288}, 3^{1536}, 2^{20}, 3^{2304}, 2^{10}, 1^{576}, 1^{3456}, 5^{256}, 4^{768}, 1^{4608}, 1^{512}, 1^{2688},$

k = 6 n = 16 #29091 $1^{144}, 2^{1785}, 3^{5093}, 4^{15853}, 5^{6109}, 6^{107},$

AUT: $3^{120}, 369^{32}, 4337^{4}, 12928^{1}, 2006^{8}, 112^{64}, 886^{16}, 138^{24}, 7565^{2}, 182^{12}, 189^{6}, 9^{3}, 2^{60}, 110^{48}, 2^{240}, 4^{72}, 7^{36}, 15^{144}, 4^{720}, 1^{11520}, 48^{96}, 38^{192}, 59^{128}, 22^{384}, 7^{576}, 4^{288}, 9^{10}, 2^{20}, 2^{1152}, 3^{2688}, 7^{768}, 9^{256}, 5^{1536}, 2^{512}, 1^{1024}, 1^{36864}, 1^{2304}, 1^{5376}, 1^{20160},$

k = 6 n = 17 #70600 $1^{145}, 2^{1984}, 3^{6529}, 4^{29025}, 5^{30232}, 6^{2683}, 7^{2},$

AUT: $61^{96}, 1295^{16}, 528^{32}, 3241^{8}, 7896^{4}, 43^{192}, 39086^{1}, 17198^{2}, 1^{60}, 283^{12}, 275^{6}, 3^{120}, 9^{3}, 150^{24}, 136^{48}, 2^{240}, 39^{384}, 1^{11520}, 11^{36}, 4^{720}, 2^{1440}, 183^{64}, 64^{128}, 5^{576}, 4^{20}, 10^{10}, 12^{144}, 6^{1152}, 3^{72}, 2^{8064}, 2^{40320}, 10^{768}, 10^{256}, 6^{1536}, 3^{288}, 3^{512}, 5^{2688}, 1^{1024}, 1^{12288}, 2^{2304}, 1^{9216}, 1^{5376}, 1^{1344}, 1^{322560},$

k = 6 n = 18 #164705 $1^{129}, 2^{1989}, 3^{7384}, 4^{41375}, 5^{83378}, 6^{30380}, 7^{69}, 8^{1},$

AUT: $691^{32}, 12992^{4}, 4957^{8}, 1884^{16}, 106222^{1}, 36124^{2}, 1^{2160}, 167^{48}, 395^{6}, 1^{120}, 176^{24}, 434^{12}, 9^{144}, 15^{3}, 73^{96}, 1^{18}, 11^{72}, 2^{720}, 1^{240}, 52^{384}, 285^{64}, 14^{768}, 9^{36}, 1^{432}, 2^{1440}, 70^{128}, 51^{192}, 4^{20}, 2^{10}, 1^{8064}, 1^{120960}, 6^{2688}, 7^{1152}, 14^{256}, 2^{4608}, 4^{288}, 5^{1536}, 2^{10752}, 2^{2304}, 4^{512}, 2^{1024}, 1^{3072}, 1^{6144}, 2^{576}, 1^{12288}, 1^{1344}, 1^{21504}, 1^{40320}, 1^{43008},$

k = 6 n = 19 #366089 $1^{113}, 2^{1799}, 3^{7428}, 4^{49110}, 5^{149230}, 6^{152012}, 7^{6384}, 8^{13},$

AUT: $20260^{4}, 69626^{2}, 262984^{1}, 202^{48}, 7181^{8}, 255^{24}, 2673^{16}, 547^{12}, 635^{6}, 45^{3}, 875^{32}, 9^{72}, 27^{36}, 365^{64}, 18^{768}, 14^{144}, 75^{192}, 106^{128}, 48^{384}, 77^{96}, 3^{2688}, 2^{4608}, 5^{2304}, 19^{256}, 2^{5376}, 6^{1536}, 2^{10752}, 4^{1152}, 8^{576}, 2^{1024}, 4^{512}, 1^{6144}, 4^{3072}, 2^{9216}, 1^{36864}, 1^{1344}, 1^{43008},$

k = 6 n = 20 #770232 $1^{91}, 2^{1489}, 3^{6685}, 4^{50328}, 5^{197792}, 6^{410022}, 7^{103234}, 8^{591},$

AUT: $222^{48}, 9974^{8}, 713^{12}, 30132^{4}, 597418^{1}, 124828^{2}, 3559^{16}, 45^{3}, 855^{6}, 1208^{32}, 7^{20}, 106^{96}, 313^{24}, 1^{60}, 12^{144}, 85^{192}, 424^{64}, 29^{256}, 3^{120}, 6^{10}, 2^{240}, 31^{36}, 5^{72}, 6^{576}, 127^{128}, 50^{384}, 17^{768}, 5^{2304}, 1^{4608}, 1^{320}, 1^{32256}, 2^{5376}, 5^{1152}, 5^{288}, 1^{23040}, 8^{1536}, 8^{512}, 3^{1024}, 5^{3072}, 1^{184320}, 2^{6144}, 2^{9216}, 1^{18432}, 1^{36864}, 1^{2688}, 1^{10752},$

k = 6 n = 21 #1528188 $1^{67}, 2^{1124}, 3^{5429}, 4^{45409}, 5^{212276}, 6^{692388}, 7^{549030}, 8^{22465},$

AUT: $339^{24}, 43735^{4}, 11^{144}, 13198^{8}, 1252490^{1}, 208745^{2}, 2^{1008}, 1^{336}, 27^{10}, 4572^{16}, 130^{96}, 9^{20}, 1^{42}, 1114^{6}, 969^{12}, 15^{72}, 274^{48}, 7^{120}, 74^{192}, 501^{64}, 42^{256}, 1573^{32}, 143^{128}, 2^{720}, 18^{36}, 1^{14}, 1^{432}, 1^{5040}, 50^{3}, 1^{7}, 1^{240}, 1^{18}, 55^{384}, 7^{1152}, 17^{768}, 3^{4608}, 1^{160}, 1^{320}, 1^{32256}, 7^{288}, 1^{3456}, 1^{23040}, 3^{1920}, 10^{1536}, 14^{512}, 3^{1024}, 1^{61440}, 6^{3072}, 3^{6144}, 2^{576}, 2^{2304}, 1^{12288}, 1^{10752},$

k = 6 n = 22 #2852541 $1^{50}, 2^{775}, 3^{3975}, 4^{36487}, 5^{193925}, 6^{842032}, 7^{1450844}, 8^{324268}, 9^{185},$

AUT: $5766^{16}, 17127^{8}, 60955^{4}, 2432863^{1}, 329077^{2}, 8^{20}, 403^{24}, 1140^{12}, 1585^{6}, 19^{144}, 34^{36}, 333^{48}, 18^{72}, 27^{10}, 4^{240}, 3^{720}, 132^{96}, 1909^{32}, 178^{128}, 10^{576}, 86^{192}, 530^{64}, 26^{768}, 7^{120}, 131^{3}, 1^{160}, 6^{1152}, 53^{384}, 46^{256}, 10^{1536}, 3^{1920}, 2^{288}, 19^{512}, 1^{320}, 9^{1024}, 2^{3072}, 1^{61440}, 2^{9216}, 2^{6144}, 2^{2304}, 2^{3840}, 2^{336}, 2^{1008}, 1^{2688}, 1^{11520}, 2^{2048}, 1^{12288},$

k = 6 n = 23 #5002807 $1^{34}, 2^{499}, 3^{2630}, 4^{26215}, 5^{154664}, 6^{814153}, 7^{2300240}, 8^{1664427}, 9^{39924}, 10^{21},$

AUT: $21449^{8}, 83374^{4}, 490210^{2}, 498^{24}, 6971^{16}, 8^{20}, 4393015^{1}, 1957^{6}, 1317^{12}, 398^{48}, 1^{60}, 4^{240}, 9^{576}, 85^{192}, 612^{64}, 10^{72}, 2092^{32}, 23^{768}, 6^{10}, 154^{96}, 6^{1152}, 37^{36}, 130^{3}, 2^{120}, 11^{288}, 22^{144}, 68^{384}, 201^{128}, 64^{256}, 10^{1536}, 21^{512}, 1^{320}, 11^{1024}, 5^{3072}, 2^{4608}, 1^{184320}, 1^{18432}, 2^{3840}, 6^{336}, 2^{672}, 1^{168}, 1^{2688}, 1^{1344}, 1^{720}, 1^{6144}, 1^{21504}, 2^{2304}, 2^{2048}, 1^{36864},$

k = 6 n = 24 #8239576 $1^{21}, 2^{300}, 3^{1598}, 4^{16898}, 5^{108892}, 6^{662780}, 7^{2547025}, 8^{4146456}, 9^{753455}, 10^{2151},$

AUT: $568^{24}, 25963^{8}, 8042^{16}, 110615^{4}, 694082^{2}, 7391999^{1}, 2315^{6}, 21^{72}, 1576^{12}, 17^{144}, 2324^{32}, 193^{96}, 464^{48}, 657^{64}, 7^{1152}, 101^{192}, 77^{384}, 107^{3}, 26^{36}, 11^{288}, 1^{3456}, 224^{128}, 27^{512}, 80^{256}, 11^{1536}, 12^{1024}, 5^{3072}, 2^{4608}, 19^{768}, 1^{110592}, 8^{336}, 2^{1008}, 2^{672}, 4^{2688}, 3^{168}, 2^{6144}, 1^{1344}, 1^{516096}, 2^{2048}, 1^{18432}, 1^{2304}, 1^{12288}, 1^{21504},$

k = 6 n = 25 #12742312 $1^{14}, 2^{168}, 3^{894}, 4^{9856}, 5^{68014}, 6^{467576}, 7^{2200244}, 8^{6046697}, 9^{3846346}, 10^{102500}, 11^{3},$

AUT: $936121^{2}, 11614023^{1}, 142700^{4}, 30218^{8}, 2^{720}, 664^{24}, 720^{64}, 124^{192}, 2640^{32}, 8809^{16}, 249^{128}, 229^{96}, 479^{48}, 25^{144}, 5^{240}, 50^{384}, 2904^{6}, 1774^{12}, 251^{3}, 11^{10}, 13^{20}, 4^{120}, 46^{36}, 20^{72}, 92^{256}, 9^{576}, 29^{512}, 5^{1152}, 13^{1536}, 35^{768}, 1^{60}, 10^{1024}, 5^{3072}, 1^{36864}, 1^{1008}, 1^{4032}, 1^{1344}, 9^{336}, 4^{168}, 3^{6144}, 3^{2688}, 1^{73728}, 1^{21504}, 2^{64512}, 2^{2048}, 1^{4096}, 2^{5376},$

k = 6 n = 26 #18504121 $1^{9}, 2^{91}, 3^{461}, 4^{5231}, 5^{37832}, 6^{289667}, 7^{1581979}, 8^{6043766}, 9^{8991699}, 10^{1552760}, 11^{625}, 12^{1},$

AUT: $17063872^{1}, 1209286^{2}, 174692^{4}, 35171^{8}, 6^{240}, 2799^{32}, 9525^{16}, 815^{64}, 121^{192}, 2^{1920}, 283^{128}, 721^{24}, 3380^{6}, 46^{10}, 2^{160}, 13^{20}, 251^{3}, 1993^{12}, 30^{144}, 502^{48}, 14^{120}, 45^{36}, 242^{96}, 5^{288}, 14^{72}, 55^{384}, 7^{576}, 3^{1152}, 101^{256}, 11^{1536}, 32^{512}, 30^{768}, 9^{1024}, 3^{4608}, 1^{36864}, 2^{1344}, 2^{2048}, 2^{720}, 1^{672}, 2^{168}, 6^{336}, 8^{3072}, 1^{11520}, 3^{6144}, 2^{9216}, 2^{2688}, 2^{18432}, 1^{24576}, 2^{2304}, 1^{4096}, 2^{5376},$

k = 6 n = 27 #25238326 $1^{5}, 2^{48}, 3^{228}, 4^{2542}, 5^{18815}, 6^{158581}, 7^{978152}, 8^{4649716}, 9^{11986035}, 10^{7306592}, 11^{137552}, 12^{60},$

AUT: $23474513^{1}, 9941^{16}, 881^{64}, 263^{96}, 3^{1440}, 506^{48}, 1495852^{2}, 2263^{12}, 1^{51840}, 757^{24}, 205102^{4}, 60^{384}, 40388^{8}, 2984^{32}, 14^{120}, 3782^{6}, 172^{3}, 2^{160}, 46^{10}, 128^{192}, 13^{20}, 1^{18}, 333^{128}, 111^{256}, 18^{72}, 31^{36}, 11^{288}, 21^{144}, 4^{1152}, 13^{1536}, 1^{1296}, 33^{512}, 13^{1024}, 1^{110592}, 4^{240}, 2^{1344}, 1^{4608},$

$22^{768}, 2^{576}, 2^{2048}, 1^{672}, 2^{720}, 10^{3072}, 3^{1920}, 2^{6144}, 3^{336}, 4^{18432}, 2^{9216}, 1^{8064}, 1^{73728}, 1^{12288}, 1^{1008}$,

k = 6 n = 28 #32339615 $1^{3}, 2^{24}, 3^{108}, 4^{1158}, 5^{8418}, 6^{76982}, 7^{527718}, 8^{2950388}, 9^{10727929}, 10^{15580010}, 11^{2461104}, 12^{5773}$,

AUT: $522^{48}, 10735^{16}, 64^{384}, 1^{40320}, 2^{1344}, 30265761^{1}, 44768^{8}, 982^{64}, 3033^{32}, 1773116^{2}, 255^{96}, 807^{24}, 1^{84}, 4412^{6}, 6^{120}, 231537^{4}, 2390^{12}, 350^{3}, 11^{10}, 12^{20}, 1^{168}, 1^{42}, 132^{192}, 3^{1440}, 54^{36}, 14^{72}, 2^{14}, 1^{7}, 361^{128}, 26^{144}, 114^{256}, 35^{512}, 17^{1536}, 6^{288}, 23^{768}, 1^{60}, 6^{576}, 10^{3072}, 3^{1008}, 4^{240}, 1^{4032}, 12^{1024}, 3^{2048}, 3^{1152}, 4^{9216}, 4^{2304}, 1^{258048}, 3^{6144}, 2^{12288}, 1^{18432}, 1^{73728}, 1^{36864}$,

k = 6 n = 29 #38939167 $1^{2}, 2^{12}, 3^{48}, 4^{500}, 5^{3434}, 6^{33277}, 7^{249752}, 8^{1599581}, 9^{7288293}, 10^{18621717}, 11^{10888743}, 12^{253807}, 13^{1}$,

AUT: $3187^{32}, 1089^{64}, 36598319^{1}, 2013735^{2}, 47782^{8}, 253402^{4}, 4745^{6}, 2530^{12}, 353^{3}, 335^{128}, 11483^{16}, 549^{48}, 119^{192}, 51^{36}, 5^{1152}, 37^{768}, 234^{96}, 852^{24}, 37^{144}, 11^{72}, 6^{288}, 72^{384}, 127^{256}, 31^{512}, 22^{1536}, 4^{576}, 1^{1008}, 3^{336}, 3^{2048}, 1^{8064}, 1^{2688}, 9^{1024}, 6^{9216}, 1^{86016}, 2^{64512}, 9^{3072}, 4^{4608}, 3^{6144}, 3^{2304}, 2^{4096}, 1^{24576}, 1^{12288}$,

k = 6 n = 30 #44065939 $1^{1}, 2^{6}, 3^{22}, 4^{204}, 5^{1300}, 6^{12941}, 7^{103923}, 8^{752544}, 9^{4045844}, 10^{14761621}, 11^{20977052}, 12^{3409270}, 13^{1210}, 14^{1}$,

AUT: $41528250^{1}, 2193485^{2}, 2653^{12}, 50525^{8}, 266854^{4}, 897^{24}, 216^{96}, 12104^{16}, 4827^{6}, 352^{128}, 22^{144}, 1^{60}, 218^{3}, 1^{18}, 30^{36}, 3227^{32}, 24^{72}, 14^{10}, 1^{360}, 14^{20}, 578^{48}, 2^{432}, 1140^{64}, 116^{192}, 77^{384}, 2^{240}, 8^{288}, 42^{768}, 126^{256}, 2^{720}, 26^{1536}, 2^{23040}, 10^{1152}, 6^{336}, 4^{120}, 30^{512}, 2^{320}, 2^{168}, 3^{2688}, 8^{3072}, 3^{2048}, 9^{1024}, 3^{21504}, 4^{2304}, 1^{645120}, 4^{6144}, 1^{86016}, 2^{9216}, 5^{4608}, 2^{4096}, 1^{73728}, 1^{18432}, 1^{12288}, 1^{43008}$,

k = 6 n = 31 #46875729 $1^{1}, 2^{3}, 3^{10}, 4^{83}, 5^{463}, 6^{4595}, 7^{38207}, 8^{309021}, 9^{1904532}, 10^{8800701}, 11^{22261786}, 12^{13298702}, 13^{257551}, 14^{73}, 15^{1}$,

AUT: $44233207^{1}, 2289529^{2}, 272777^{4}, 67^{384}, 2611^{12}, 52687^{8}, 52^{36}, 5109^{6}, 12099^{16}, 975^{24}, 390^{3}, 569^{48}, 3372^{32}, 51^{10}, 16^{20}, 3^{60}, 24^{72}, 1074^{64}, 236^{96}, 94^{192}, 430^{128}, 4^{240}, 10^{120}, 2^{11520}, 4^{576}, 57^{768}, 27^{144}, 21^{1536}, 2^{23040}, 113^{256}, 2^{720}, 6^{336}, 2^{1008}, 26^{512}, 2^{320}, 4^{160}, 5^{168}, 4^{1152}, 2^{1920}, 12^{3072}, 7^{2304}, 3^{2048}, 3^{2688}, 1^{645120}, 4^{21504}, 1^{9999360}, 4^{6144}, 4^{9216}, 1^{258048}, 10^{1024}, 3^{288}, 1^{4608}, 2^{12288}, 1^{516096}, 1^{1344}, 2^{64512}, 1^{18432}, 1^{36864}, 1^{43008}, 1^{322560}$,

k = 6 n = 32 #46875730 $1^{1}, 2^{2}, 3^{4}, 4^{33}, 5^{159}, 6^{1511}, 7^{12525}, 8^{111163}, 9^{772132}, 10^{4256108}, 11^{15460946}, 12^{22540719}, 13^{3711510}, 14^{8915}, 15^{1}, 16^{1}$,

AUT: $44233207^{1}, 2289529^{2}, 272777^{4}, 52687^{8}, 12099^{16}, 5109^{6}, 390^{3}, 2611^{12}, 975^{24}, 16^{20}, 3^{60}, 51^{10}, 24^{72}, 569^{48}, 3372^{32}, 1074^{64}, 94^{192}, 236^{96}, 430^{128}, 67^{384}, 10^{120}, 2^{11520}, 4^{576}, 52^{36}, 57^{768}, 4^{240}, 2^{23040}, 27^{144}, 2^{720}, 2^{1008}, 113^{256}, 1^{4608}, 4^{160}, 2^{320}, 21^{1536}, 26^{512}, 4^{1152}, 3^{288}, 5^{168}, 6^{336}, 2^{1920}, 7^{2304}, 12^{3072}, 3^{2048}, 3^{2688}, 1^{322560}, 1^{319979520}, 4^{21504}, 1^{43008}, 2^{64512}, 10^{1024}, 1^{36864}, 4^{6144}, 4^{9216}, 1^{18432}, 2^{12288}, 1^{516096}, 1^{1344}, 1^{258048}, 1^{645120}, 1^{9999360}$,

k = 6 n = 33 #44065940 $2^{1}, 3^{2}, 4^{12}, 5^{53}, 6^{473}, 7^{3711}, 8^{35258}, 9^{271045}, 10^{1734360}, 11^{8001319}, 12^{20823754}, 13^{12861939}, 14^{334005}, 15^{7}, 16^{1}$,

AUT: $41528250^{1}, 2193485^{2}, 2653^{12}, 50525^{8}, 266854^{4}, 897^{24}, 216^{96}, 12104^{16}, 4827^{6}, 352^{128}, 22^{144}, 1^{60}, 218^{3}, 1^{18}, 30^{36}, 3227^{32}, 24^{72}, 14^{10}, 1^{360}, 14^{20}, 578^{48}, 2^{432}, 1140^{64}, 116^{192}, 77^{384}, 2^{240}, 8^{288}, 42^{768}, 126^{256}, 2^{720}, 26^{1536}, 2^{23040}, 10^{1152}, 6^{336}, 4^{120}, 30^{512}, 2^{320}, 2^{168}, 3^{2688}, 8^{3072}, 3^{2048}, 9^{1024}, 3^{21504}, 4^{2304}, 1^{645120}, 4^{6144}, 1^{86016}, 2^{9216}, 5^{4608}, 2^{4096}, 1^{73728}, 1^{18432}, 1^{12288}, 1^{43008}, 1^{10321920}$,

k = 6 n = 34 #38939168 $3^{1}, 4^{5}, 5^{17}, 6^{143}, 7^{1023}, 8^{9986}, 9^{82630}, 10^{605121}, 11^{3334202}, 12^{12509164}, 13^{18927652}, 14^{3467119}, 15^{2103}, 16^{2}$,

AUT: $3187^{32}, 1089^{64}, 36598319^{1}, 2013735^{2}, 47782^{8}, 253402^{4}, 4745^{6}, 2530^{12}, 353^{3}, 335^{128}, 11483^{16}, 549^{48}, 119^{192}, 51^{36}, 5^{1152}, 37^{768}, 234^{96}, 852^{24}, 37^{144}, 11^{72}, 6^{288}, 72^{384}, 127^{256}, 31^{512}, 22^{1536}, 4^{576}, 1^{1008}, 3^{336}, 3^{2048}, 1^{8064}, 1^{2688}, 9^{1024}, 6^{9216}, 1^{86016}, 2^{64512}, 9^{3072}, 4^{4608}, 3^{6144}, 3^{2304}, 2^{4096}, 1^{24576}, 1^{12288}, 1^{688128}$,

k = 6 n = 35 #32339617 $4^{2}, 5^{6}, 6^{41}, 7^{269}, 8^{2586}, 9^{22018}, 10^{181878}, 11^{1162030}, 12^{5570756}, 13^{15016920}, 14^{10119401}, 15^{263582}, 16^{128}$,

AUT: $522^{48}, 10735^{16}, 64^{384}, 1^{40320}, 2^{1344}, 30265761^{1}, 44768^{8}, 982^{64}, 3033^{32}, 1773116^{2}, 255^{96}, 807^{24}, 1^{84}, 4412^{6}, 6^{120}, 231537^{4}, 2390^{12}, 350^{3}, 11^{10}, 12^{20}, 1^{168}, 1^{42}, 132^{192}, 3^{1440}, 54^{36}, 14^{72}, 2^{14}, 1^{7}, 361^{128}, 26^{144}, 114^{256}, 35^{512}, 17^{1536}, 6^{288}, 23^{768}, 1^{60}, 6^{576}, 10^{3072}, 3^{1008}, 4^{240}, 1^{4032}, 12^{1024}, 3^{2048}, 3^{1152}, 4^{9216}, 4^{2304}, 1^{258048}, 3^{6144}, 2^{12288}, 1^{18432}, 2^{73728}, 1^{36864}, 1^{2064384}$,

k = 6 n = 36 #25238329 $5^{2}, 6^{13}, 7^{67}, 8^{629}, 9^{5215}, 10^{47345}, 11^{343890}, 12^{1987406}, 13^{7692725}, 14^{12581053}, 15^{2571507}, 16^{8477}$,

AUT: $23474513^{1}, 9941^{16}, 881^{64}, 263^{96}, 3^{1440}, 506^{48}, 1495852^{2}, 2263^{12}, 1^{51840}, 757^{24}, 205102^{4}, 60^{384}, 40388^{8}, 2984^{32}, 14^{120}, 3782^{6}, 172^{3}, 2^{160}, 46^{10}, 128^{192}, 13^{20}, 1^{18}, 333^{128}, 111^{256}, 18^{72}, 31^{36}, 11^{288}, 21^{144}, 4^{1152}, 13^{1536}, 1^{1296}, 33^{512}, 13^{1024}, 1^{110592}, 4^{240}, 2^{1344}, 1^{4608}, 22^{768}, 2^{576}, 2^{2048}, 1^{672}, 2^{720}, 10^{3072}, 3^{1920}, 2^{6144}, 3^{336}, 4^{18432}, 2^{9216}, 1^{8064}, 2^{73728}, 2^{12288}, 1^{1008}, 1^{294912}$,

k = 6 n = 37 #18504126 $6^{4}, 7^{18}, 8^{147}, 9^{1127}, 10^{10822}, 11^{86930}, 12^{589841}, 13^{2907817}, 14^{8398926}, 15^{6276930}, 16^{231562}, 17^{2}$,

AUT: $17063872^{1}, 1209286^{2}, 174692^{4}, 35171^{8}, 6^{240}, 2799^{32}, 9525^{16}, 815^{64}, 121^{192}, 2^{1920}, 283^{128}, 721^{24}, 3380^{6}, 46^{10}, 2^{160}, 13^{20}, 251^{3}, 1993^{12}, 30^{144}, 502^{48}, 14^{120}, 45^{36}, 242^{96}, 5^{288}, 14^{72}, 55^{384}, 7^{576}, 3^{1152}, 101^{256}, 11^{1536}, 32^{512}, 30^{768}, 9^{1024}, 3^{4608}, 1^{36864}, 2^{1344}, 2^{2048}, 2^{720}, 1^{672}, 2^{168}, 6^{336}, 8^{3072}, 1^{11520}, 4^{6144}, 2^{9216}, 2^{2688}, 2^{18432}, 1^{24576}, 2^{2304}, 1^{4096}, 2^{5376}, 1^{98304}, 1^{12288}, 1^{61440}, 1^{3840}$,

k = 6 n = 38 #12742321 $7^{5}, 8^{35}, 9^{228}, 10^{2228}, 11^{18916}, 12^{147985}, 13^{875855}, 14^{3619918}, 15^{6480540}, 16^{1594288}, 17^{2322}, 18^{1}$,

AUT: $936121^{2}, 11614023^{1}, 142700^{4}, 30218^{8}, 2^{720}, 664^{24}, 720^{64}, 124^{192}, 2640^{32}, 8809^{16}, 249^{128}, 229^{96}, 479^{48}, 25^{144}, 5^{240}, 50^{384}, 2904^{6}, 1774^{12}, 251^{3}, 11^{10}, 13^{20}, 4^{120}, 46^{36}, 20^{72}, 92^{256}, 9^{576}, 29^{512}, 6^{1152}, 14^{1536}, 35^{768}, 1^{60}, 10^{1024}, 5^{3072}, 2^{36864}, 1^{1008}, 1^{4032}, 1^{1344}, 9^{336}, 4^{168}, 4^{6144}, 3^{2688}, 1^{73728}, 1^{21504}, 2^{64512}, 2^{2048}, 2^{4096}, 2^{5376}, 1^{294912}, 1^{24576}, 1^{3840}, 1^{23040}$,

k = 6 n = 39 #8239590 $8^{8}, 9^{46}, 10^{424}, 11^{3608}, 12^{31719}, 13^{217988}, 14^{1151075}, 15^{3576454}, 16^{3119211}, 17^{138931}, 18^{126}$,

AUT: $568^{24}, 25963^{8}, 8042^{16}, 110615^{4}, 694082^{2}, 7391999^{1}, 2315^{6}, 21^{72}, 1576^{12}, 17^{144}, 2324^{32}, 193^{96}, 464^{48}, 657^{64}, 7^{1152}, 101^{192}, 78^{384}, 107^{3}, 26^{36}, 11^{288}, 1^{3456}, 224^{128}, 28^{512}, 80^{256}, 14^{1536}, 12^{1024}, 5^{3072}, 3^{4608}, 19^{768}, 1^{110592}, 8^{336}, 2^{1008}, 2^{672}, 4^{2688}, 3^{168}, 2^{6144}, 1^{1344}, 1^{516096}, 2^{2048}, 1^{18432}, 3^{2304}, 2^{12288}, 1^{21504}, 1^{2064384}, 2^{4096}, 1^{24576}, 1^{86016}$,

k = 6 n = 40 #5002828 $9^{9}, 10^{80}, 11^{620}, 12^{5914}, 13^{45593}, 14^{291394}, 15^{1275318}, 16^{2598637}, 17^{780507}, 18^{4756}$,

AUT: $21449^{8}, 83374^{4}, 490210^{2}, 498^{24}, 6971^{16}, 8^{20}, 4393015^{1}, 1957^{6}, 1317^{12}, 398^{48}, 1^{60}, 4^{240}, 9^{576}, 85^{192}, 612^{64}, 10^{72}, 2092^{32}, 24^{768}, 6^{10}, 154^{96}, 6^{1152}, 37^{36}, 130^{3}, 2^{120}, 11^{288}, 22^{144}, 69^{384}, 201^{128}, 67^{256}, 15^{1536}, 23^{512}, 1^{320}, 12^{1024}, 5^{3072}, 2^{4608}, 1^{184320}, 1^{18432}, 2^{3840}, 6^{336}, 2^{672}, 1^{168}, 1^{2688}, 1^{1344}, 1^{720}, 1^{6144}, 1^{21504}, 2^{2304}, 2^{2048}, 1^{36864}, 1^{24576}, 2^{4096}, 1^{86016}, 2^{12288}, 1^{688128}, 1^{5376}$,

k = 6 n = 41 #2852575 $10^{15}, 11^{101}, 12^{987}, 13^{8138}, 14^{61001}, 15^{336072}, 16^{1163772}, 17^{1201416}, 18^{81056}, 19^{17}$,

AUT: $5766^{16}, 17127^{8}, 60955^{4}, 2432863^{1}, 329077^{2}, 8^{20}, 403^{24}, 1140^{12}, 1585^{6}, 19^{144}, 34^{36}, 333^{48}, 18^{72}, 27^{10}, 4^{240}, 3^{720}, 132^{96}, 1909^{32}, 181^{128}, 10^{576}, 86^{192}, 530^{64}, 28^{768}, 7^{120}, 131^{3}, 1^{160}, 6^{1152}, 54^{384}, 52^{256}, 15^{1536}, 3^{1920}, 2^{288}, 21^{512}, 1^{320}, 11^{1024}, 2^{3072}, 1^{61440}, 2^{9216}, 4^{6144}, 3^{2304}, 2^{3840}, 2^{336}, 2^{1008}, 1^{2688}, 1^{11520}, 2^{2048}, 2^{12288}, 1^{36864}, 1^{4096}, 1^{24576}, 1^{98304}, 2^{10752}, 1^{43008}, 2^{4608}$,

k = 6 n = 42 #1528238 $11^{16}, 12^{155}, 13^{1266}, 14^{10822}, 15^{70540}, 16^{340387}, 17^{791009}, 18^{312279}, 19^{1756}, 20^{8}$,

AUT: $339^{24}, 43735^{4}, 11^{144}, 13198^{8}, 1252490^{1}, 208745^{2}, 2^{1008}, 1^{336}, 27^{10}, 4572^{16}, 130^{96}, 9^{20}, 1^{42}, 1114^{6}, 969^{12}, 15^{72}, 274^{48}, 7^{120}, 74^{192}, 503^{64}, 52^{256}, 1573^{32}, 148^{128}, 2^{720}, 18^{36}, 1^{14}, 1^{432}, 1^{5040}, 50^{3}, 1^{7}, 1^{240}, 1^{18}, 58^{384}, 7^{1152}, 18^{768}, 4^{4608}, 1^{160}, 2^{320}, 2^{32256}, 7^{288}, 1^{3456}, 2^{23040}, 3^{1920}, 17^{1536}, 18^{512}, 6^{1024}, 3^{61440}, 6^{3072}, 7^{6144}, 2^{576}, 3^{2304}, 3^{12288}, 2^{10752}, 1^{32768}$,

k = 6 n = 43 #770299 $12^{23}, 13^{180}, 14^{1672}, 15^{12285}, 16^{74527}, 17^{282084}, 18^{364340}, 19^{35075}, 20^{113}$,

AUT: $222^{48}, 9974^{8}, 713^{12}, 30132^{4}, 597418^{1}, 124828^{2}, 3559^{16}, 45^{3}, 855^{6}, 1208^{32}, 7^{20}, 106^{96}, 313^{24}, 1^{60}, 12^{144}, 86^{192}, 430^{64}, 38^{256}, 3^{120}, 6^{10}, 2^{240}, 31^{36}, 5^{72}, 6^{576}, 138^{128}, 53^{384}, 21^{768}, 5^{2304}, 1^{4608}, 2^{320}, 1^{32256}, 2^{5376}, 5^{1152}, 5^{288}, 13^{1536}, 2^{3840}, 1^{23040}, 17^{512}, 8^{1024}, 7^{3072}, 1^{184320}, 4^{6144}, 2^{9216}, 1^{18432}, 1^{36864}, 1^{2688}, 1^{10752}, 1^{24576}, 1^{98304}, 2^{12288}, 2^{2048}, 1^{61440}$,

k = 6 n = 44 #366180 $13^{24}, 14^{235}, 15^{1830}, 16^{13250}, 17^{66862}, 18^{185142}, 19^{97025}, 20^{1811}, 21^{1}$,

AUT: $20260^{4}, 69626^{2}, 262984^{1}, 202^{48}, 7181^{8}, 255^{24}, 2673^{16}, 547^{12}, 635^{6}, 45^{3}, 876^{32}, 9^{72}, 27^{36}, 377^{64}, 23^{768}, 14^{144}, 77^{192}, 119^{128}, 57^{384}, 77^{96}, 12^{1536}, 3^{2688}, 2^{4608}, 5^{2304}, 30^{256}, 2^{5376}, 2^{10752}, 6^{1152}, 8^{576}, 16^{512}, 4^{6144}, 8^{1024}, 6^{3072}, 2^{9216}, 2^{36864}, 1^{1344}, 1^{43008}, 1^{294912}, 1^{24576}, 1^{4096}, 2^{2048}, 1^{73728}$,

k = 6 n = 45 #164818 $14^{30}, 15^{244}, 16^{2013}, 17^{12114}, 18^{52562}, 19^{84032}, 20^{13796}, 21^{26}, 22^{1}$,

AUT: $694^{32}, 12992^{4}, 4957^{8}, 1884^{16}, 106222^{1}, 36124^{2}, 1^{2160}, 167^{48}, 395^{6}, 1^{120}, 176^{24}, 434^{12}, 9^{144}, 15^{3}, 73^{96}, 1^{18}, 11^{72}, 2^{720}, 1^{240}, 63^{384}, 300^{64}, 15^{768}, 9^{36}, 1^{432}, 2^{1440}, 91^{128}, 55^{192}, 4^{20}, 2^{10}, 16^{512}, 30^{256}, 13^{1536}, 1^{8064}, 1^{120960}, 6^{2688}, 8^{1152}, 3^{4608}, 4^{288}, 2^{10752}, 9^{1024}, 4^{6144}, 2^{2304}, 3^{3072}, 2^{576}, 1^{12288}, 1^{1344}, 1^{21504}, 1^{40320}, 1^{43008}, 1^{98304}, 2^{18432}, 1^{4096}, 2^{2048}, 1^{36864}, 1^{24576}$,

k = 6 n = 46 #70729 $15^{30}, 16^{280}, 17^{1822}, 18^{10383}, 19^{32473}, 20^{24973}, 21^{759}, 22^{9}$,

AUT: $61^{96}, 1295^{16}, 531^{32}, 3241^{8}, 7896^{4}, 46^{192}, 39086^{1}, 17198^{2}, 1^{60}, 283^{12}, 275^{6}, 3^{120}, 9^{3}, 150^{24}, 136^{48}, 2^{240}, 48^{384}, 1^{11520}, 11^{36}, 4^{720}, 2^{1440}, 207^{64}, 91^{128}, 5^{576}, 4^{20}, 10^{10}, 12^{144}, 6^{1152}, 3^{72}, 18^{512}, 16^{1536}, 26^{256}, 12^{768}, 2^{8064}, 2^{40320}, 3^{288}, 5^{2688}, 3^{2048}, 7^{1024}, 2^{12288}, 2^{2304}, 1^{9216}, 1^{5376}, 1^{1344}, 1^{688128}, 1^{322560}, 3^{6144}, 2^{3072}, 2^{4096}, 1^{24576}, 1^{86016}$,

$\mathbf{k = 6\ n = 47}$ #29236 $16^{38}, 17^{243}, 18^{1652}, 19^{7378}, 20^{15691}, 21^{4153}, 22^{80}, 23^{1},$

AUT: $3^{120}, 372^{32}, 4337^{4}, 12928^{1}, 2006^{8}, 142^{64}, 886^{16}, 138^{24}, 7565^{2}, 182^{12}, 189^{6}, 9^{3}, 2^{60}, 110^{48}, 2^{240}, 4^{72}, 7^{36}, 15^{144}, 4^{720}, 1^{11520}, 48^{96}, 2^{23040}, 45^{192}, 85^{128}, 26^{384}, 7^{576}, 4^{288}, 9^{10}, 2^{20}, 1^{4608}, 18^{1536}, 2^{320}, 18^{512}, 27^{256}, 3^{1152}, 3^{2688}, 9^{768}, 3^{2048}, 5^{1024}, 1^{36864}, 1^{2304}, 1^{5376}, 1^{43008}, 1^{10321920}, 1^{20160}, 4^{6144}, 1^{12288}, 1^{18432}, 2^{4096}, 1^{73728}, 1^{86016}, 1^{645120},$

$\mathbf{k = 6\ n = 48}$ #11780 $17^{30}, 18^{234}, 19^{1235}, 20^{4740}, 21^{5039}, 22^{498}, 23^{3}, 24^{1},$

AUT: $243^{32}, 1^{360}, 1220^{8}, 1^{120}, 2101^{4}, 5^{1152}, 552^{16}, 58^{96}, 3756^{1}, 3094^{2}, 132^{12}, 93^{48}, 88^{6}, 96^{64}, 91^{24}, 2^{3}, 6^{72}, 1^{240}, 2^{720}, 3^{36}, 4^{144}, 29^{192}, 2^{23040}, 16^{1536}, 76^{128}, 21^{384}, 7^{288}, 2^{20}, 3^{2304}, 2^{10}, 6^{768}, 17^{512}, 2^{320}, 23^{256}, 1^{576}, 1^{3456}, 2^{4608}, 3^{2048}, 1^{86016}, 4^{6144}, 1^{645120}, 1^{2688}, 4^{1024}, 2^{4096}, 1^{73728}, 1^{18432}, 1^{12288}, 1^{43008}, 1^{30965760},$

$\mathbf{k = 6\ n = 49}$ #4708 $18^{30}, 19^{179}, 20^{931}, 21^{2303}, 22^{1233}, 23^{30}, 24^{2},$

AUT: $310^{16}, 1^{56448}, 62^{24}, 945^{4}, 1^{168}, 673^{8}, 1126^{2}, 928^{1}, 152^{32}, 73^{48}, 52^{96}, 6^{576}, 6^{1152}, 76^{12}, 27^{192}, 72^{64}, 36^{6}, 1^{14}, 2^{72}, 6^{36}, 13^{1536}, 50^{128}, 18^{384}, 16^{512}, 19^{256}, 5^{768}, 2^{2304}, 5^{144}, 3^{2048}, 3^{288}, 1^{4032}, 1^{4608}, 6^{1024}, 3^{6144}, 1^{86016}, 2^{4096}, 1^{24576}, 2^{3072}, 1^{12288}, 1^{2064384},$

$\mathbf{k = 6\ n = 50}$ #1907 $19^{24}, 20^{153}, 21^{553}, 22^{978}, 23^{190}, 24^{9},$

AUT: $49^{64}, 81^{32}, 165^{16}, 6^{1152}, 24^{192}, 2^{4032}, 47^{24}, 378^{4}, 360^{2}, 16^{6}, 180^{1}, 329^{8}, 36^{12}, 5^{144}, 54^{48}, 41^{96}, 3^{72}, 1^{720}, 4^{576}, 2^{20}, 6^{36}, 30^{128}, 19^{384}, 9^{1536}, 19^{256}, 12^{512}, 1^{23040}, 3^{768}, 1^{3840}, 7^{1024}, 1^{36864}, 1^{4608}, 1^{240}, 2^{2048}, 1^{672}, 3^{6144}, 1^{24576}, 2^{18432}, 1^{2304}, 2^{3072}, 1^{4096}, 1^{294912},$

$\mathbf{k = 6\ n = 51}$ #791 $20^{23}, 21^{99}, 22^{338}, 23^{288}, 24^{43},$

AUT: $43^{32}, 27^{64}, 85^{16}, 24^{96}, 16^{192}, 5^{144}, 2^{1344}, 7^{1152}, 1^{20}, 137^{4}, 88^{2}, 16^{12}, 24^{1}, 6^{6}, 144^{8}, 23^{24}, 36^{48}, 1^{36}, 3^{72}, 2^{288}, 1^{432}, 1^{576}, 2^{240}, 2^{120}, 2^{10}, 12^{512}, 16^{384}, 17^{128}, 12^{256}, 7^{1536}, 2^{1920}, 1^{23040}, 1^{3840}, 2^{3072}, 6^{1024}, 5^{768}, 1^{36864}, 1^{672}, 1^{336}, 2^{2048}, 2^{73728}, 3^{6144}, 1^{4096}, 1^{884736},$

$\mathbf{k = 6\ n = 52}$ #340 $21^{16}, 22^{77}, 23^{152}, 24^{91}, 25^{4},$

AUT: $9^{96}, 1^{576}, 12^{24}, 49^{16}, 18^{32}, 11^{192}, 12^{128}, 8^{12}, 1^{4032}, 44^{8}, 2^{1008}, 41^{4}, 17^{2}, 1^{6}, 1^{120}, 2^{10}, 8^{144}, 26^{48}, 2^{288}, 11^{64}, 5^{1536}, 1^{720}, 9^{256}, 9^{384}, 9^{512}, 1^{320}, 5^{1024}, 3^{3840}, 2^{1920}, 1^{61440}, 2^{3072}, 4^{768}, 2^{6144}, 1^{1344}, 2^{336}, 1^{8064}, 1^{1152}, 1^{12288}, 2^{2048}, 1^{294912}, 1^{73728}, 1^{36864},$

$\mathbf{k = 6\ n = 53}$ #155 $22^{15}, 23^{43}, 24^{76}, 25^{19}, 26^{2},$

AUT: $5^{32}, 19^{16}, 4^{96}, 2^{576}, 7^{12}, 12^{8}, 6^{64}, 6^{128}, 5^{24}, 5^{144}, 17^{48}, 3^{1008}, 8^{4}, 1^{120}, 5^{384}, 1^{320}, 1^{4608}, 1^{32256}, 2^{288}, 4^{192}, 10^{256}, 1^{23040}, 1^{72}, 7^{1536}, 4^{512}, 3^{1024}, 1^{3840}, 3^{6144}, 1^{61440}, 1^{2304}, 1^{336}, 1^{768}, 1^{10752}, 1^{12288}, 1^{2688}, 1^{98304}, 1^{36864}, 1^{18432}, 1^{184320},$

$\mathbf{k = 6\ n = 54}$ #72 $23^{9}, 24^{31}, 25^{24}, 26^{8},$

AUT: $9^{48}, 1^{1296}, 2^{192}, 2^{96}, 3^{32}, 2^{12}, 2^{72}, 4^{128}, 3^{24}, 5^{16}, 3^{144}, 2^{4608}, 1^{8}, 2^{384}, 2^{768}, 6^{256}, 5^{1536}, 2^{10752}, 2^{288}, 2^{512}, 2^{1024}, 1^{2304}, 2^{12288}, 1^{6144}, 1^{336}, 1^{294912}, 1^{43008}, 1^{18432}, 1^{110592}, 1^{73728},$

$\mathbf{k = 6\ n = 55}$ #35 $24^{8}, 25^{15}, 26^{11}, 27^{1},$

AUT: $1^{720}, 4^{144}, 3^{48}, 1^{240}, 1^{24}, 1^{1152}, 1^{192}, 5^{1536}, 1^{768}, 1^{384}, 3^{256}, 1^{72}, 2^{512}, 3^{12288}, 1^{1024}, 1^{96}, 1^{36864}, 1^{2064384}, 1^{5376}, 1^{73728}, 1^{258048},$

$\mathbf{k = 6\ n = 56}$ #19 $25^{5}, 26^{10}, 27^{3}, 28^{1},$

AUT: $2^{144}, 1^{240}, 1^{5040}, 1^{720}, 2^{2304}, 3^{1536}, 1^{512}, 1^{384}, 1^{4608}, 1^{73728}, 2^{12288}, 1^{258048}, 1^{36864}, 1^{14450688},$

$\mathbf{k = 6\ n = 57}$ #10 $26^{4}, 27^{4}, 28^{2},$

AUT: $1^{1152}, 1^{23040}, 1^{720}, 1^{3840}, 1^{110592}, 1^{12288}, 1^{18432}, 1^{73728}, 1^{1536}, 1^{2064384},$

$\mathbf{k = 6\ n = 58}$ #5 $27^{2}, 28^{3},$

AUT: $1^{18432}, 1^{184320}, 1^{3840}, 1^{688128}, 1^{36864},$

$\mathbf{k = 6\ n = 59}$ #3 $28^{2}, 29^{1},$

AUT: $1^{516096}, 1^{2064384}, 1^{36864},$

$\mathbf{k = 6\ n = 60}$ #2 $29^{1}, 30^{1},$

AUT: $1^{30965760}, 1^{516096},$

$\mathbf{k = 6\ n = 61}$ #1 $30^{1},$

AUT: $1^{10321920},$

$\mathbf{k = 6\ n = 62}$ #1 $31^{1},$

AUT: $1^{319979520},$

$\mathbf{k = 6\ n = 63}$ #1 $32^{1},$

AUT: $1^{20158709760},$

Appendix 2 Number of the self-orthogonal projective codes in dimension 5 and 6.

$\mathbf{k = 5\ n = 12}$ #1 $4^{1},$
$\mathbf{k = 5\ n = 15}$ #2 $4^{1}, 6^{1},$
$\mathbf{k = 5\ n = 16}$ #3 $4^{1}, 6^{1}, 8^{1},$
$\mathbf{k = 5\ n = 19}$ #1 $8^{1},$
$\mathbf{k = 5\ n = 20}$ #1 $8^{1},$
$\mathbf{k = 5\ n = 23}$ #1 $8^{1},$
$\mathbf{k = 5\ n = 24}$ #1 $12^{1},$
$\mathbf{k = 5\ n = 31}$ #1 $16^{1},$
$\mathbf{k = 6\ n = 12}$ #1 $4^{1},$
$\mathbf{k = 6\ n = 13}$ #1 $4^{1},$
$\mathbf{k = 6\ n = 14}$ #2 $4^{2},$
$\mathbf{k = 6\ n = 15}$ #2 $4^{2},$
$\mathbf{k = 6\ n = 16}$ #5 $4^{4}, 6^{1},$
$\mathbf{k = 6\ n = 17}$ #4 $4^{3}, 6^{1},$
$\mathbf{k = 6\ n = 18}$ #5 $4^{2}, 6^{2}, 8^{1},$
$\mathbf{k = 6\ n = 19}$ #12 $4^{8}, 6^{4},$
$\mathbf{k = 6\ n = 20}$ #17 $4^{8}, 6^{6}, 8^{3},$
$\mathbf{k = 6\ n = 21}$ #14 $4^{3}, 6^{6}, 8^{5},$
$\mathbf{k = 6\ n = 22}$ #19 $4^{3}, 6^{6}, 8^{10},$
$\mathbf{k = 6\ n = 23}$ #40 $4^{6}, 6^{9}, 8^{25},$
$\mathbf{k = 6\ n = 24}$ #52 $4^{4}, 6^{9}, 8^{38}, 10^{1},$
$\mathbf{k = 6\ n = 25}$ #42 $4^{2}, 6^{5}, 8^{30}, 10^{5},$
$\mathbf{k = 6\ n = 26}$ #50 $4^{2}, 6^{4}, 8^{31}, 10^{12}, 12^{1},$
$\mathbf{k = 6\ n = 27}$ #89 $4^{2}, 6^{5}, 8^{43}, 10^{34}, 12^{5},$
$\mathbf{k = 6\ n = 28}$ #103 $4^{1}, 6^{3}, 8^{34}, 10^{54}, 12^{11},$
$\mathbf{k = 6\ n = 29}$ #75 $6^{1}, 8^{16}, 10^{44}, 12^{14},$
$\mathbf{k = 6\ n = 30}$ #85 $6^{1}, 8^{12}, 10^{38}, 12^{34},$
$\mathbf{k = 6\ n = 31}$ #133 $4^{1}, 6^{2}, 8^{15}, 10^{45}, 12^{70},$
$\mathbf{k = 6\ n = 32}$ #134 $4^{1}, 6^{1}, 8^{9}, 10^{30}, 12^{87}, 14^{5}, 16^{1},$
$\mathbf{k = 6\ n = 33}$ #85 $8^{3}, 10^{10}, 12^{63}, 14^{9},$
$\mathbf{k = 6\ n = 34}$ #75 $8^{1}, 10^{6}, 12^{45}, 14^{22}, 16^{1},$
$\mathbf{k = 6\ n = 35}$ #103 $8^{2}, 10^{6}, 12^{42}, 14^{46}, 16^{7},$
$\mathbf{k = 6\ n = 36}$ #89 $8^{1}, 10^{2}, 12^{26}, 14^{49}, 16^{11},$
$\mathbf{k = 6\ n = 37}$ #50 $10^{1}, 12^{8}, 14^{27}, 16^{14},$
$\mathbf{k = 6\ n = 38}$ #42 $10^{1}, 12^{4}, 14^{16}, 16^{21},$
$\mathbf{k = 6\ n = 39}$ #53 $8^{1}, 12^{6}, 14^{13}, 16^{33},$
$\mathbf{k = 6\ n = 40}$ #41 $12^{3}, 14^{5}, 16^{31}, 18^{2},$
$\mathbf{k = 6\ n = 41}$ #19 $14^{2}, 16^{11}, 18^{6},$
$\mathbf{k = 6\ n = 42}$ #14 $14^{1}, 16^{5}, 18^{6}, 20^{2},$
$\mathbf{k = 6\ n = 43}$ #18 $12^{1}, 16^{8}, 18^{7}, 20^{2},$
$\mathbf{k = 6\ n = 44}$ #13 $16^{4}, 18^{4}, 20^{5},$
$\mathbf{k = 6\ n = 45}$ #5 $18^{2}, 20^{3},$
$\mathbf{k = 6\ n = 46}$ #4 $18^{1}, 20^{3},$
$\mathbf{k = 6\ n = 47}$ #8 $16^{3}, 20^{4}, 22^{1},$
$\mathbf{k = 6\ n = 48}$ #5 $20^{3}, 22^{1}, 24^{1},$
$\mathbf{k = 6\ n = 49}$ #2 $22^{1}, 24^{1},$
$\mathbf{k = 6\ n = 50}$ #1 $22^{1},$
$\mathbf{k = 6\ n = 51}$ #2 $20^{1}, 24^{1},$
$\mathbf{k = 6\ n = 52}$ #1 $24^{1},$
$\mathbf{k = 6\ n = 55}$ #1 $24^{1},$
$\mathbf{k = 6\ n = 56}$ #1 $28^{1},$
$\mathbf{k = 6\ n = 63}$ #1 $32^{1},$

Appendix 3 Number of the codes with dual distance ≥ 4 in dimension 5 and 6.

$\mathbf{k = 5\ n = 5}$ #1 $1^{1},$
$\mathbf{k = 5\ n = 6}$ #3 $1^{2}, 2^{1},$
$\mathbf{k = 5\ n = 7}$ #3 $1^{1}, 2^{2},$
$\mathbf{k = 5\ n = 8}$ #4 $1^{1}, 2^{3},$
$\mathbf{k = 5\ n = 9}$ #5 $1^{1}, 2^{2}, 3^{2},$
$\mathbf{k = 5\ n = 10}$ #4 $2^{1}, 3^{1}, 4^{2},$
$\mathbf{k = 5\ n = 11}$ #2 $3^{1}, 4^{1},$
$\mathbf{k = 5\ n = 12}$ #2 $4^{2},$
$\mathbf{k = 5\ n = 13}$ #1 $5^{1},$
$\mathbf{k = 5\ n = 14}$ #1 $6^{1},$
$\mathbf{k = 5\ n = 15}$ #1 $7^{1},$
$\mathbf{k = 5\ n = 16}$ #1 $8^{1},$
$\mathbf{k = 6\ n = 6}$ #1 $1^{1},$
$\mathbf{k = 6\ n = 7}$ #4 $1^{3}, 2^{1},$
$\mathbf{k = 6\ n = 8}$ #7 $1^{3}, 2^{4},$
$\mathbf{k = 6\ n = 9}$ #12 $1^{4}, 2^{8},$
$\mathbf{k = 6\ n = 10}$ #24 $1^{5}, 2^{16}, 3^{3},$
$\mathbf{k = 6\ n = 11}$ #34 $1^{4}, 2^{18}, 3^{11}, 4^{1},$
$\mathbf{k = 6\ n = 12}$ #43 $1^{2}, 2^{14}, 3^{17}, 4^{10},$
$\mathbf{k = 6\ n = 13}$ #47 $1^{2}, 2^{8}, 3^{18}, 4^{19},$
$\mathbf{k = 6\ n = 14}$ #49 $1^{1}, 2^{7}, 3^{10}, 4^{28}, 5^{3},$
$\mathbf{k = 6\ n = 15}$ #44 $1^{1}, 2^{3}, 3^{9}, 4^{20}, 5^{10}, 6^{1},$
$\mathbf{k = 6\ n = 16}$ #48 $1^{1}, 2^{3}, 3^{4}, 4^{21}, 5^{13}, 6^{6},$
$\mathbf{k = 6\ n = 17}$ #40 $1^{1}, 2^{2}, 3^{3}, 4^{9}, 5^{16}, 6^{8}, 7^{1},$
$\mathbf{k = 6\ n = 18}$ #33 $2^{1}, 3^{1}, 4^{6}, 5^{7}, 6^{17}, 7^{1},$
$\mathbf{k = 6\ n = 19}$ #25 $3^{1}, 4^{2}, 5^{5}, 6^{10}, 7^{7},$
$\mathbf{k = 6\ n = 20}$ #24 $4^{2}, 5^{2}, 6^{8}, 7^{7}, 8^{5},$

$\mathbf{k=6\ n=21}$ #16 5^2, 6^3, 7^7, 8^4,
$\mathbf{k=6\ n=22}$ #15 6^3, 7^3, 8^8, 9^1,
$\mathbf{k=6\ n=23}$ #9 7^3, 8^3, 9^3,
$\mathbf{k=6\ n=24}$ #8 8^4, 9^2, 10^2,
$\mathbf{k=6\ n=25}$ #5 9^3, 10^2,
$\mathbf{k=6\ n=26}$ #4 10^3, 11^1,
$\mathbf{k=6\ n=27}$ #2 11^2,
$\mathbf{k=6\ n=28}$ #2 12^2,
$\mathbf{k=6\ n=29}$ #1 13^1,
$\mathbf{k=6\ n=30}$ #1 14^1,
$\mathbf{k=6\ n=31}$ #1 15^1,
$\mathbf{k=6\ n=32}$ #1 16^1,

Appendix 4 Projective codes with transitive automorphism groups.

b000000000000000	[3,3,1];	$A_1 = 3, A_2 = 3, A_3 = 1,$	AUT: 6
b400000000000000	[4,3,2];	$A_2 = 6, A_4 = 1,$	AUT: 24
b800000000000000	[4,4,1];	$A_1 = 4, A_2 = 6, A_3 = 4, A_4 = 1,$	AUT: 24
b804000000000000	[5,4,2];	$A_2 = 10, A_4 = 5,$	AUT: 120
b808000000000000	[5,5,1];	$A_1 = 5, A_2 = 10, A_3 = 10, A_4 = 5, A_5 = 1,$	AUT: 120
b808000400000000	[6,5,2];	$A_2 = 15, A_4 = 15, A_6 = 1,$	AUT: 720
b808000800000000	[6,6,1];	$A_1 = 6, A_2 = 15, A_3 = 20, A_4 = 15, A_5 = 6, A_6 = 1,$	AUT: 720
b808000800000004	[7,6,2];	$A_2 = 21, A_4 = 35, A_6 = 7,$	AUT: 5040
f888000800080000	[9,6,2];	$A_2 = 9, A_4 = 27, A_6 = 27,$	AUT: 1296
f880000000000000	[6,4,2];	$A_2 = 6, A_4 = 9,$	AUT: 72
bc40000000000000	[6,4,2];	$A_2 = 3, A_3 = 8, A_4 = 3, A_6 = 1,$	AUT: 48
bc08000800000800	[8,6,2];	$A_2 = 12, A_4 = 38, A_6 = 12, A_8 = 1,$	AUT: 1152
bc18200880000804	[12,6,4];	$A_4 = 18, A_6 = 24, A_8 = 21,$	AUT: 144
fd08080808000800	[12,6,3];	$A_3 = 8, A_4 = 6, A_6 = 16, A_7 = 24, A_8 = 9,$	AUT: 1152
ff08080808080800	[14,6,4];	$A_4 = 14, A_8 = 49,$	AUT: 56448
bc08001800208000	[10,6,3];	$A_3 = 10, A_4 = 15, A_5 = 12, A_6 = 15, A_7 = 10, A_{10} = 1,$	AUT: 120
bc88005804208120	[15,6,6];	$A_6 = 30, A_8 = 15, A_{10} = 18,$	AUT: 360
fc08101a88208000	[15,6,5];	$A_5 = 12, A_6 = 10, A_8 = 15, A_9 = 20, A_{10} = 6,$	AUT: 120
bc41000000000000	[7,4,3];	$A_3 = 7, A_4 = 7, A_7 = 1,$	AUT: 168
bc48400000000000	[8,5,2];	$A_2 = 4, A_4 = 22, A_6 = 4, A_8 = 1,$	AUT: 384
bc48000900014002	[12,6,4];	$A_4 = 6, A_5 = 24, A_6 = 16, A_8 = 9, A_9 = 8,$	AUT: 1152
bc48011000000000	[9,5,3];	$A_3 = 6, A_4 = 9, A_5 = 9, A_6 = 6, A_9 = 1,$	AUT: 72
bc48010800101002	[12,6,4];	$A_4 = 15, A_6 = 32, A_8 = 15, A_{12} = 1,$	AUT: 120
bc48014904115006	[18,6,8];	$A_8 = 45, A_{12} = 18,$	AUT: 2160
ff48090a10501112	[21,6,8];	$A_8 = 21, A_{12} = 42,$	AUT: 1008
bee8150810501112	[21,6,8];	$A_8 = 21, A_{12} = 42,$	AUT: 336
fd48010810501112	[18,6,6];	$A_6 = 12, A_8 = 9, A_{10} = 36, A_{12} = 6,$	AUT: 144
bc48011400000000	[10,5,4];	$A_4 = 15, A_6 = 15, A_{10} = 1,$	AUT: 720
bc48011840001000	[12,6,3];	$A_3 = 4, A_4 = 3, A_5 = 12, A_6 = 24, A_7 = 12,$ $A_8 = 3, A_9 = 4, A_{12} = 1,$	AUT: 144
bc48011840021084	[15,6,6];	$A_6 = 25, A_8 = 30, A_{10} = 3, A_{12} = 5,$	AUT: 720
bc48011c40001440	[15,6,5];	$A_5 = 6, A_6 = 10, A_7 = 15, A_8 = 15, A_9 = 10,$ $A_{10} = 6, A_{15} = 1,$	AUT: 720
bc48011c40001442	[16,6,6];	$A_6 = 16, A_8 = 30, A_{10} = 16, A_{16} = 1,$	AUT: 11520
fc48155c400216e2	[24,6,10];	$A_{10} = 24, A_{12} = 12, A_{14} = 24, A_{16} = 3,$	AUT: 1152
fd58111cc80214c2	[24,6,10];	$A_{10} = 24, A_{12} = 12, A_{14} = 24, A_{16} = 3,$	AUT: 384
fdd8311cd80214c6	[28,6,12];	$A_{12} = 28, A_{16} = 35,$	AUT: 40320
ff58191cc84215c2	[28,6,12];	$A_{12} = 28, A_{16} = 35,$	AUT: 1344
bd5c011fe8095442	[27,6,12];	$A_{12} = 36, A_{16} = 27,$	AUT: 51840
be48851840401030	[18,6,6];	$A_6 = 6, A_8 = 27, A_{10} = 18, A_{12} = 12,$	AUT: 432
fdd83118d8001000	[21,6,6];	$A_6 = 7, A_{10} = 21, A_{12} = 35,$	AUT: 5040
fc48155600000000	[15,5,6];	$A_6 = 10, A_8 = 15, A_{10} = 6,$	AUT: 720
bc48400840000000	[10,6,2];	$A_2 = 5, A_4 = 10, A_5 = 32, A_6 = 10, A_8 = 5, A_{10} = 1,$	AUT: 3840
bc48400801000801	[12,6,4];	$A_4 = 15, A_6 = 32, A_8 = 15, A_{12} = 1,$	AUT: 384
bc48400840000003	[12,6,4];	$A_4 = 15, A_6 = 32, A_8 = 15, A_{12} = 1,$	AUT: 23040
bc48400300000000	[10,5,4];	$A_4 = 10, A_5 = 16, A_8 = 5,$	AUT: 1920
bd58c80000000000	[12,5,4];	$A_4 = 6, A_6 = 16, A_8 = 9,$	AUT: 48
fdd8d80000000000	[15,5,5];	$A_5 = 6, A_8 = 15, A_9 = 10,$	AUT: 720
bf48484000000000	[12,5,4];	$A_4 = 6, A_6 = 16, A_8 = 9,$	AUT: 1152
bc43000000000000	[8,4,4];	$A_4 = 14, A_8 = 1,$	AUT: 1344
bdd9c548505a8261	[28,6,12];	$A_{12} = 28, A_{16} = 35,$	AUT: 84

bc4b430000000000	[12,5,4];	$A_4 = 3, A_6 = 24, A_8 = 3, A_{12} = 1,$	AUT: 2304
bc4b411841209802	[20,6,8];	$A_8 = 15, A_{10} = 32, A_{12} = 15, A_{20} = 1,$	AUT: 120
bf7bc1585129d802	[30,6,12];	$A_{12} = 15, A_{16} = 45, A_{20} = 3,$	AUT: 360
bc4b430840303000	[18,6,6];	$A_6 = 6, A_8 = 9, A_9 = 32, A_{10} = 9, A_{12} = 6, A_{18} = 1,$	AUT: 2304
bc4b430840303003	[20,6,8];	$A_8 = 15, A_{10} = 32, A_{12} = 15, A_{20} = 1,$	AUT: 23040
bccbc309483a3214	[27,6,12];	$A_{12} = 27, A_{14} = 27, A_{18} = 9,$	AUT: 1296
bcebe3494c383013	[30,6,12];	$A_{12} = 5, A_{14} = 30, A_{16} = 15, A_{18} = 10, A_{20} = 3,$	AUT: 720
bccfc70bf4303003	[30,6,12];	$A_{12} = 10, A_{15} = 32, A_{16} = 15, A_{20} = 6,$	AUT: 23040
bc4b430843000000	[16,6,4];	$A_4 = 4, A_8 = 54, A_{12} = 4, A_{16} = 1,$	AUT: 36864
bc4b4308430000b4	[20,6,8];	$A_8 = 10, A_{10} = 48, A_{16} = 5,$	AUT: 184320
bc7b73084b484000	[24,6,8];	$A_8 = 6, A_{12} = 48, A_{16} = 9,$	AUT: 1536
bf7b7b487b484000	[30,6,10];	$A_{10} = 6, A_{15} = 32, A_{16} = 15, A_{18} = 10,$	AUT: 23040
bcff430843084300	[24,6,8];	$A_8 = 6, A_{12} = 48, A_{16} = 9,$	AUT: 110592
bc4b433000000000	[14,5,6];	$A_6 = 7, A_7 = 16, A_8 = 7, A_{14} = 1,$	AUT: 2688
bc4b43b000000000	[15,5,7];	$A_7 = 15, A_8 = 15, A_{15} = 1,$	AUT: 20160
bc4b433801909800	[21,6,7];	$A_7 = 3, A_9 = 7, A_{10} = 21, A_{11} = 21, A_{12} = 7,$ $A_{14} = 3, A_{21} = 1,$	AUT: 1008
bc4b433843309801	[24,6,10];	$A_{10} = 12, A_{12} = 38, A_{14} = 12, A_{24} = 1,$	AUT: 1152
bc4b43b400000000	[16,5,8];	$A_8 = 30, A_{16} = 1,$	AUT: 322560
bc4b43bc43b40000	[24,6,8];	$A_8 = 3, A_{12} = 56, A_{16} = 3, A_{24} = 1,$	AUT: 516096
bc4b43bc43b4b400	[28,6,12];	$A_{12} = 7, A_{14} = 48, A_{16} = 7, A_{28} = 1,$	AUT: 258048
bc4b43bc43b4bc40	[30,6,14];	$A_{14} = 15, A_{15} = 32, A_{16} = 15, A_{30} = 1,$	AUT: 645120
bc4b43bc43b4bc41	[31,6,15];	$A_{15} = 31, A_{16} = 31, A_{31} = 1,$	AUT: 9999360
bc4b43bc43b4bc43	[32,6,16];	$A_{16} = 62, A_{32} = 1,$	AUT: 319979520
fd5f57f500000000	[24,5,12];	$A_{12} = 28, A_{16} = 3,$	AUT: 64512
ff7f77f700000000	[28,5,14];	$A_{14} = 24, A_{16} = 7,$	AUT: 64512
fffff7f700000000	[30,5,15];	$A_{15} = 16, A_{16} = 15,$	AUT: 322560
fffffff700000000	[31,5,16];	$A_{16} = 31,$	AUT: 9999360
bd5f577100000000	[21,5,10];	$A_{10} = 21, A_{12} = 7, A_{14} = 3,$	AUT: 1008
bc7b730300000000	[18,5,8];	$A_8 = 9, A_9 = 16, A_{12} = 6,$	AUT: 1152
bf7b7b4000000000	[20,5,8];	$A_8 = 5, A_{10} = 16, A_{12} = 10,$	AUT: 1920
fd57000000000000	[12,4,6];	$A_6 = 12, A_8 = 3,$	AUT: 576
ff77000000000000	[14,4,7];	$A_7 = 8, A_8 = 7,$	AUT: 1344
fff7000000000000	[15,4,8];	$A_8 = 15,$	AUT: 20160
bd54000000000000	[9,4,4];	$A_4 = 9, A_6 = 6,$	AUT: 72
fdd0000000000000	[10,4,4];	$A_4 = 5, A_6 = 10,$	AUT: 120
f500000000000000	[6,3,3];	$A_3 = 4, A_4 = 3,$	AUT: 24
f700000000000000	[7,3,4];	$A_4 = 7,$	AUT: 168
8ffffffffffffff7	[60,6,30];	$A_{30} = 48, A_{32} = 15,$	AUT: 30965760
00f7f7f7f7f7f7f7	[49,6,24];	$A_{24} = 49, A_{28} = 14,$	AUT: 56448
43b7feb6fbeeaff1	[45,6,20];	$A_{20} = 18, A_{24} = 45,$	AUT: 2160
00b7f6f5efafeee5	[42,6,20];	$A_{20} = 42, A_{24} = 21,$	AUT: 1008
0227cee327fdeb31	[35,6,16];	$A_{16} = 35, A_{20} = 28,$	AUT: 40320
42a3fee017f6abb5	[36,6,16];	$A_{16} = 27, A_{20} = 36,$	AUT: 51840
0000fff7fff7fff7	[45,6,22];	$A_{22} = 45, A_{24} = 15, A_{30} = 3,$	AUT: 120960
0000bff7bff7bff7	[42,6,20];	$A_{20} = 21, A_{21} = 32, A_{24} = 7, A_{28} = 3,$	AUT: 32256
0000bcf7e7efeeb3	[36,6,16];	$A_{16} = 18, A_{18} = 24, A_{20} = 18, A_{24} = 3,$	AUT: 144
00b4bcf7bcffff43	[40,6,16];	$A_{16} = 5, A_{20} = 48, A_{24} = 10,$	AUT: 184320
0000bcf7bcf7bcf7	[36,6,16];	$A_{16} = 9, A_{18} = 48, A_{24} = 6,$	AUT: 110592
0008fffffffffff7	[48,6,24];	$A_{24} = 60, A_{32} = 3,$	AUT: 30965760
08fffffffffffff7	[56,6,28];	$A_{28} = 56, A_{32} = 7,$	AUT: 14450688
fffffffffffffff7	[63,6,32];	$A_{32} = 63,$	AUT: 20158709760
fffffffffffffff3	[62,6,31];	$A_{31} = 32, A_{32} = 31,$	AUT: 319979520
fffffffffffffff1	[61,6,30];	$A_{30} = 16, A_{31} = 32, A_{32} = 15,$	AUT: 10321920

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