On q-ary optimal equitable symbol weight codes

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OC2013 Albena, Bulgaria, September 6-12, 2013

Outline

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Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ be an arbitrary word over Q. Denote by $\xi_a(\boldsymbol{x})$ the number of times the symbol $a \in Q$ occurs in $\boldsymbol{x}$, i.e.

$$\xi_a(\boldsymbol{x}) = |\{j : \ x_j = a, \ j = 1, 2, \ldots, n\}|.$$

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Definition 1.

A code C over Q we call *equitable symbol weight code*, if every its codeword has equitable symbol weight.

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Several optimal infinite families of such codes were constructed by Chee-Kiah-Ling-Wang (2012) and also by Dai-Wang-Yin (2013).
In the paper Dai-Wang-Yin (2013) a family of q-ary optimal equitable symbol weight codes was constructed with the following parameters:

$$n = q^2 - 1, \;\; M = q^2, \;\; d = q(q-1), \tag{1}$$

for any q equal to a power of odd prime number.

In this paper we construct, using the other approach, equitable symbol weight codes with parameters (1) for any prime power q.

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$$n, \;\; M = n(q-1), \;\; d = n(q-1)/q, \qquad (2)$$

where q divides n, and n is such, that there exists a difference matrix of size $n \times n$ over the alphabet Q.

It is well known (see, for example, (Semakov-Zaitzev-Zinoviev, 1969) or (Bogdanova-Zinoviev-Todorov, 2007) and references there) that for any prime power q, can be easily constructed optimal equidistant q-ary codes with the following parameters:

length	n	$=$	$q^2 - 1$
minimum distance	d	$=$	$q(q-1)$
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$$\begin{array}{ccc} \text{length} & n & = & q^2 - 1 \\ \text{minimum distance} & d & = & q(q-1) \\ \text{cardinality} & M & = & q^2. \end{array}$$

These codes are not equitable symbol weight, but it is possible to transform their such that they become equitable symbol weight codes without missing the property to be equidistant.

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Let A be a matrix of size $q \times q$ of the form

$$\begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 1 & \dots & 1 \\ . & . & \dots & . \\ q-1 & q-1 & \dots & q-1 \end{bmatrix},$$

and let $L_1, L_2, ..., L_{q-1}$ be a set of $q-1$ Latin squares of order q over Q with the following property: the pairwise distance between any two rows of different squares is equal to $q-1$ (it is clear that the pairwise distance between any two rows of one square is equal to q).

The rows of the following matrix of size $q^2 \times (q^2-1)$ form an equidistant code with parameters ($n = q^2-1$, $d = q(q-1)$ and $M = q^2$) mentioned above:

$$\left[\begin{array}{ccc|cccc} A & \cdots & A & \boldsymbol{e}_0 & \boldsymbol{e}_1 & \cdots & \boldsymbol{e}_{q-2} \\ L_1 & \cdots & L_1 & \boldsymbol{e}_1 & \boldsymbol{e}_2 & \cdots & \boldsymbol{e}_{q-1} \\ \cdots & & \cdots & \cdots & \cdots & & \cdots \\ L_{q-1} & \cdots & L_{q-1} & \boldsymbol{e}_{q-1} & \boldsymbol{e}_0 & \cdots & \boldsymbol{e}_{q-3} \end{array}\right],$$

where $\boldsymbol{e}_i$ is the column-vector $(i\, i\, \ldots\, i)^t$.

$$\left[\begin{array}{cc} A & \cdots\ A \\ \hline L_1 & \cdots\ L_1 \\ \cdots & \cdots \\ L_{q-1} & \cdots\ L_{q-1} \end{array}\right],$$

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Apart of the layer formed by the matrix A the other matrix is of the equal symbol weight type.

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steps of the transformation:
1) to correct the first layer of matrices A by adding a proper vector

$$\boldsymbol{a} = (\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_{q-1}), \ \ \boldsymbol{a}_j = (0, j, j, \ldots, j)$$

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2) to correct the lower part of the matrix by proper permutations independently for every Latin square of every layer

Main result 1

Theorem 1. *For any prime power q there exists an optimal equitable symbol weight equidistant q-ary code with the following parameters*:

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Note once more that for the case of odd q this result has been obtained in (Dai-Wang-Yin, 2013) using the other approach.

Consider an example of our construction for the case $q = 4$. Let $Q_4 = \{0, 1, 2, 3\}$, where $1 = \alpha^0$, $2 = \alpha$, $3 = \alpha^2$, and the element α is the primitive element of the field $\mathbf{F}_4$ such that $\alpha^2 = \alpha + 1$.

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Let C be the following matrix, formed by the codewords of equidistant $(n = 5, M = 16, d = 4)$ code over $Q = \{0, 1, 2, 3\}$:

$$C = \left[\begin{array}{c|c} A & \boldsymbol{e}_0 \\ L_1 & \boldsymbol{e}_1 \\ L_2 & \boldsymbol{e}_2 \\ L_3 & \boldsymbol{e}_3 \end{array}\right], \quad A = \begin{bmatrix} 0\,0\,0\,0 \\ 1\,1\,1\,1 \\ 2\,2\,2\,2 \\ 3\,3\,3\,3 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0\,1\,2\,3 \\ 1\,0\,3\,2 \\ 2\,3\,0\,1 \\ 3\,2\,1\,0 \end{bmatrix}, L_2 = \begin{bmatrix} 0\,2\,3\,1 \\ 1\,3\,2\,0 \\ 2\,0\,1\,3 \\ 3\,1\,0\,2 \end{bmatrix}, L_3 = \begin{bmatrix} 0\,3\,1\,2 \\ 1\,2\,0\,3 \\ 2\,1\,3\,0 \\ 3\,0\,2\,1 \end{bmatrix},$$

where $\boldsymbol{e}_i = (i\,i\,i\,i)^t$, $i = 0, 1, 2, 3$.

Construct the equidistant $(15, 16, 12; 4)$ code E by repeting three times the given above code C

$$E = \left[\begin{array}{c|c|c|c} A & A & A & \boldsymbol{e}_0\ \boldsymbol{e}_1\ \boldsymbol{e}_2 \\ L_1 & L_1 & L_1 & \boldsymbol{e}_1\ \boldsymbol{e}_2\ \boldsymbol{e}_3 \\ L_2 & L_2 & L_2 & \boldsymbol{e}_2\ \boldsymbol{e}_3\ \boldsymbol{e}_0 \\ L_3 & L_3 & L_3 & \boldsymbol{e}_3\ \boldsymbol{e}_0\ \boldsymbol{e}_1 \end{array}\right].$$

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Define the matrix K:

$$K = \left[\begin{array}{c|c|c} A & A & A \\ L_1 & L_1 & L_1 \\ L_2 & L_2 & L_2 \\ L_3 & L_3 & L_3 \end{array}\right].$$

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Add to all rows of K the vector
$\boldsymbol{a} = (\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3)$, where $\boldsymbol{a}_1 = (0,\ 1,\ 1,\ 1)$, $\boldsymbol{a}_2 = (0,\ 2,\ 2,\ 2)$,
$\boldsymbol{a}_3 = (0,\ 3,\ 3,\ 3)$.

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Adding to this layer of the vector $\boldsymbol{a}$, we obtain the following matrices $L_1^{(1)}$, $L_1^{(2)}$ and $L_1^{(3)}$, respectively:

$$\left[\begin{array}{c} 0\,0\,3\,2 \\ \hline 1\,1\,2\,3 \\ 2\,2\,1\,0 \\ 3\,3\,0\,1 \end{array}\right], \left[\begin{array}{c} 0\,3\,0\,1 \\ 1\,2\,1\,0 \\ 2\,1\,2\,3 \\ \hline 3\,0\,3\,2 \end{array}\right], \left[\begin{array}{c} 0\,2\,1\,0 \\ 1\,3\,0\,1 \\ \hline 2\,0\,3\,2 \\ 3\,1\,2\,3 \end{array}\right].$$

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Chose the row $(0\,0\,3\,2)$ of the first matrix $L_1^{(1)}$.
This choice uniquely implies the choice of the row $(2\,1\,2\,3)$ of the second matrix $L_1^{(2)}$ and the choice of the row $(1\,3\,0\,1)$ of the third matrix $L_1^{(3)}$.

Show how to reconstruct the first nontrivial layer of the matrix K:

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As a result we obtain the vector

$$(0\,0\,3\,2,\ 2\,1\,2\,3,\ 1\,3\,0\,1)$$

which has an equitable symbol weight.

Continuing in this way we obtain the optimal equitable symbol weight equidistant 4-ary code with parameters (1) $(n = 4^2 - 1 = 15, M = 4^2 = 16, d = 4(4-1) = 12)$, whose all codewords look as follows:

0 1 1 1	0 2 2 2	0 3 3 3	0 1 2
1 0 0 0	1 3 3 3	1 2 2 2	0 1 2
2 3 3 3	2 0 0 0	2 1 1 1	0 1 2
3 2 2 2	3 1 1 1	3 0 0 0	0 1 2
0 0 3 2	2 1 2 3	1 3 0 1	1 2 3
1 1 2 3	3 0 3 2	0 2 1 0	1 2 3
2 2 1 0	0 3 0 1	3 1 2 3	1 2 3
3 3 0 1	1 2 1 0	2 0 3 2	1 2 3
0 3 2 0	2 2 3 1	1 0 1 3	2 3 0
1 2 3 1	3 3 2 0	0 1 0 2	2 3 0
2 1 0 2	0 0 1 3	3 2 3 1	2 3 0
3 0 1 3	1 1 0 2	2 3 2 0	2 3 0
0 2 0 3	2 3 1 2	1 1 3 0	3 0 1
1 3 1 2	3 2 0 3	0 0 2 1	3 0 1

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Definition 2.

Call the matrix $D(n, q)$ of size $n \times n$ over Q by the difference matrix, if the difference of any two its rows contains every symbol of the alphabet Q exactly n/q times.

Main result 2

Theorem 2. *Let integer numbers* $q \geq 2$ *and* n *be such that there exists a difference matrix* $D(n, q)$ *over the alphabet* Q*. Then there exists an optimal equitable symbol weight* q*-ary code with parameters*

$$n, \; M = q(n-1), \; d = (q-1)n/q.$$

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There are $n - 1$ such rows and the pairwise distance d between any two different rows equals

$$d = n \cdot \frac{q-1}{q}$$

according to definiton of a difference matrix. Adding all these rows with vectors of length n

$$(0, \ldots, 0),\ (1, \ldots, 1),\ \ldots,\ (q-1, \ldots, q-1),$$

we obtain all together $q(n-1)$ vectors, which form our code.

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we obtain all together $q(n-1)$ vectors, which form our code.
This construction was used in
(Bassalygo-Dodunekov-Helleseth-Zinoviev, 2006) for construction of q-ary analog of binary codes, meeting the Gray-Rankin bound.

It is easy to see that this code is equitable symbol weight with two pairwise distances $(q-1)n/q$ and n (Semakov-Zaitzev-Zinoviev, 1969).

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Since every codeword has the same weight $w = (q-1)n/q$, the number of codewords is less or equal to $A_q(n, d, w)$, i.e. maximal possible number of codewords of length n, distance d on sphere of radius w.

Further

$$A_q(n,d,w) \le q \cdot A_q(n-1,d,w)$$

and

$$A_q(n-1,(q-1)n/q,(q-1)n/q) \le n-1,$$

where the last inequality follows from the following (Johnson type) bound for q-ary constant weight codes (Bassalygo, 1965):

$$A_q(n,d,w) \le \frac{\left(1-\frac{1}{q}\right)dn}{w^2-\left(1-\frac{1}{q}\right)(2w-d)n}.$$

Therefore the constructed code is optimal as equitable symbol weight code of length n with distance $d=(q-1)n/q$ (but it is not optimal as a code of length n even with the same two distances (Bassalygo-Dodunekov-Zinoviev-Helleseth, 2006)).

From Theorem 2 and the results of (Semakov-Zaitzev-Zinoviev, 1969) such optimal equitable symbol weight codes with parameters

$$n,\ M = q(n-1),\ d = (q-1)n/q.$$

exist for any $n = p^a$ and $q = p^b$, where p is a prime, and a and b, $a > b \geq 1$ are any positive integers.