New extension theorems for codes over $\mathbb{F}_q$

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Overview

- ♣ Three extension theorems are generalized.
- ♣ Some examples are given.
- ♣ A geometric method is employed to prove the results.

Contents

1. Extendability of linear codes

$\mathbb{F}_q$: the field of q elements

$\mathbb{F}_q^n = \{(a_1, a_2, ..., a_n) \mid a_1, ..., a_n \in \mathbb{F}_q\}$.

An $[n, k, d]_q$ code $\mathcal{C}$ means a k-dimensional subspace of $\mathbb{F}_q^n$ with minimum distance d,

$$d = \min\{wt(a) \mid wt(a) \neq 0,\ a \in \mathcal{C}\}.$$

We only consider non-degenerate codes

(i.e. $\not\exists i$; $c_i = 0$ for all $c = (c_1, ..., c_n) \in \mathcal{C}$).

The weight distribution (w.d.) of $\mathcal{C}$ is the list of numbers $A_i = |\{c \in \mathcal{C} \mid wt(c) = i\}|$.

The weight distribution with

$$(A_0, A_d, ..., A_i, ...) = (1, \alpha, ..., w, ...)$$

is also expressed as

$$0^1 d^\alpha \cdots i^w \cdots .$$

A linear code $\mathcal{C}$ over $\mathbb{F}_q$ is w-weight (mod q)

if

$$^{\exists}W = \{i_1, \cdots, i_w\} \subset \mathbb{Z}_q = \{0, 1, \cdots, q-1\}$$

s.t.

$$A_i > 0 \;\Rightarrow\; i \equiv i_j \pmod{q} \text{ for some } i_j \in W$$

Ex. The Golay $[11, 6, 5]_3$ code with a generator matrix

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0 \end{bmatrix}$$

has weight distribution $0^1 5^{132} 6^{132} 8^{330} 9^{110} 11^{24}$, which is 2-weight (mod 3).

$\mathcal{C}$: $[n,k,d]_q$ code with generator matrix G

$\mathcal{C}$ is extendable (to $\mathcal{C}'$) if

$[G,h]$ generates an $[n+1,k,d+1]_q$ code $\mathcal{C}'$

for some column vector h, $h^\top \in \mathbb{F}_q^k$.

$\mathcal{C}'$ is an extension of $\mathcal{C}$.

Thm 1. Every binary code with odd minimum distance is extendable.

2. Generalized extension theorems

Thm 2 (Hill-Lizak 1995).
Let $\mathcal{C}$ be an $[n,k,d]_q$ code with gcd(d,q)=1 s.t. $i \equiv 0$ or $d \pmod{q}$ for $^\forall i$ with $A_i > 0$.
Then $\mathcal{C}$ is extendable.

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Cor.

$\mathcal{C}$: an $[n,k,d]_q$ code with $d \equiv -1 \pmod q$. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \;\Rightarrow\; i \equiv 0 \text{ or } -1 \pmod q$$

3. Generalized extension theorems

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Let $\mathcal{C}$ be an $[n,k,d]_q$ code with gcd(d,q)=1 s.t. $i \equiv 0$ or d (mod q) for $^\forall i$ with $A_i > 0$. Then $\mathcal{C}$ is extendable.

Note.

- $\mathcal{C}$ is 2-weight (mod q).
- Conditon "gcd$(d,q) = 1$" is assumed.

For 3-weight (mod q) codes, the following is known:

Thm 3 (Maruta 2004, Yoshida-M 2010).
Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $q \geq 5$, $d \equiv -2$ (mod q). Then $\mathcal{C}$ is extendable if

$$A_i > 0 \;\Rightarrow\; i \equiv 0, -1, -2 \pmod{q}.$$

Note.

- $\mathrm{gcd}(d, q) = 2$ when q is even.

We give the first result for 4-weight (mod q) codes.

Thm 4.

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $q = 2^h$, $h \geq 3$, d odd. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \ \Rightarrow \ i \equiv 0, d, q/2, d + q/2 \pmod{q}.$$

Note.

- $\gcd(d, q) = 1$ since d is odd.

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$$A_i > 0 \;\Rightarrow\; i \equiv 0 \text{ or } d \pmod{q/2}.$$

Note.

- $\gcd(d,q) = 1$ since d is odd.

Example 1.

Let $\mathcal{C}_1$ be a $[100, 3, 87]_8$ code. It can be proved that all possible weights of $\mathcal{C}_1$ are $0, 87, 88, 91, 92, 95, 96$, that is,

$$A_i > 0 \ \Rightarrow \ i \equiv -1,\ 0,\ 3,\ 4 \pmod 8$$

i.e.,

$$A_i > 0 \ \Rightarrow \ i \equiv 0 \text{ or } 3 \pmod 4.$$

Hence $\mathcal{C}_1$ is extendable by Thm 4.

Example 1.

$[101, 3, 88]_8$

(a) $0^1 88^{476} 96^{35}$ (wt $\equiv 0 \pmod 8$)

(b) $0^1 88^{441} 92^{70}$ (wt $\equiv 0, 4 \pmod 8$)

$[100, 3, 87]_8$: extendable

(a-1) $0^1 87^{413} 88^{63} 95^{35}$ (wt $\equiv -1, 0$)

(a-2) $0^1 87^{420} 88^{56} 95^{28} 96^{7}$ (wt $\equiv -1, 0$)

(b-1) $0^1 87^{392} 88^{49} 91^{56} 92^{14}$ (wt $\equiv -1, 0, 3, 4$)

(b-2) $0^1 87^{378} 88^{63} 91^{70}$ (wt $\equiv -1, 0, 3$)

Example 1.

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(a-1) $0^1 87^{413} 88^{63} 95^{35}$

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(b-1) $0^1 87^{392} 88^{49} 91^{56} 92^{14}$

(b-2) $0^1 87^{378} 88^{63} 91^{70}$

Application 1.

Using the above result, we can prove the nonexistence of $[796, 4, 696]_8$ codes.

Thm 4.

Let $\mathcal{C}$ be an $[n,k,d]_q$ code with $q = 2^h$, $h \geq 3$, d odd. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \;\; \Rightarrow \;\; i \equiv 0 \text{ or } d \pmod{q/2}.$$

Application 2.

Using Thm 4, we can also prove the nonexistence of $[795,4,695]_8$ codes.

(Prove $A_j = 0$ for $j = 722, 725, 726, 733, 734$.)

Thm 2 (Hill-Lizak 1995).

Let $\mathcal{C}$ be an $[n,k,d]_q$ code with gcd(d,q)=1 s.t. $i \equiv$ 0 or d (mod q) for $^\forall i$ with $A_i > 0$. Then $\mathcal{C}$ is extendable.

Note.

- $\mathcal{C}$ is 2-weight (mod q).
- Conditon "gcd$(d,q) = 1$" is assumed.

Thm 2 (Hill-Lizak 1995).

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with gcd(d, q)=1 s.t. $i \equiv 0$ or $d \pmod{q}$ for $^\forall i$ with $A_i > 0$. Then $\mathcal{C}$ is extendable.

Note.

- $\mathcal{C}$ is 2-weight $(\bmod\ q)$.
- Conditon "gcd$(d, q) = 1$" is assumed.

Question: How about when gcd$(d, q) = 2$?

Thm 5.

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $q = 2^h$, $h \geq 3$, $\gcd(d, q) = 2$. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \ \Rightarrow \ i \equiv 0 \text{ or } d \pmod{q}.$$

Note.

- $\mathcal{C}$ is 2-weight $(\mathrm{mod}\ q)$.
- Conditon "$\gcd(d, q) = 2$" is assumed.

Example 2.

Let $\mathcal{C}_2$ be a $[73, 4, 62]_8$ code with w.d. $0^1 62^{1764} 64^{1883} 70^{252} 72^{196}$, satisfying

$$A_i > 0 \ \Rightarrow \ i \equiv 0 \text{ or } 6 \pmod 8.$$

Hence $\mathcal{C}_2$ is extendable by Thm 5.

$\mathcal{C}_2$ is from

`http://www.algorithm.uni-bayreuth.de/en/research`

`/Coding_Theory/Linear_Codes_BKW/index.html.`

Thm 5.

Let $\mathcal{C}$ be an $[n,k,d]_q$ code with $q = 2^h$, $h \geq 3$, $\gcd(d,q) = 2$. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \;\Rightarrow\; i \equiv 0 \text{ or } d \pmod{q}.$$

Note.

- $\mathcal{C}$ is 2-weight $\pmod{q}$.
- Conditon "$\gcd(d,q) = 2$" is assumed.
- Conditon "$h \geq 3$" is sharp.

Example 3 (Counterexample for $q = 4$).

$\mathcal{C}_3$: $[14, 3, 10]_4$ code with generator matrix

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & \omega & \omega & \omega & \omega & \bar{\omega} & \bar{\omega} & \bar{\omega} & \bar{\omega} \\ \omega & \bar{\omega} & \omega & \bar{\omega} & \omega & \bar{\omega} & 0 & 1 & \omega & \bar{\omega} & 0 & 1 & \omega & \bar{\omega} \end{bmatrix},$$

where $\mathbb{F}_4 = \{0, 1, \omega, \bar{\omega}\}$. The w.d. of $\mathcal{C}$ is:

$$0^1 10^{42} 12^{21} \quad (\text{wt} \equiv 0, 2 \pmod 4).$$

It can be checked that $\mathcal{C}$ is not extendable.

Thm 6 (Simonis 2000).

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $\gcd(d, q) = 1$, $q = p^h$, p prime. Then $\mathcal{C}$ is extendable if $\sum_{i \not\equiv d \pmod{p}} A_i = q^{k-1}$.

Thm 6 (Simonis 2000).

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Note.

- Conditon "gcd$(d, q) = 1$" is assumed.

Question: How about when gcd$(d, q) = 2$?

We give a generalization of Thm 6:

Thm 7.

Let h, m, t be integers with $0 \le m < t \le h$. For $q = p^h$ with prime p, every $[n, k, d]_q$ code with $\gcd(d, q) = p^m$ is extendable if

$$\sum_{i \not\equiv d \pmod{p^t}} A_i = q^{k-1}. \qquad (*)$$

Note.

- Thm 6 is the case $(m, t) = (0, 1)$.
- Conditon $(*)$ can be weakened as follows.

Thm 7'.

Let h, m, t be integers with $0 \le m < t \le h$. For $q = p^h$ with prime p, every $[n, k, d]_q$ code with $\gcd(d, q) = p^m$ is extendable if

$$\sum_{i \not\equiv d \pmod{p^t}} A_i < q^{k-1} + r(q)q^{k-3}(q-1),$$

where $q + r(q) + 1$ is the smallest size of a non-trivial blocking set in $\mathrm{PG}(2, q)$.

($r(3) = r(4) = 2$, $r(5) = 3$, $r(7) = 4$.)

Example 4.

Let $\mathcal{C}_3$ be the $[30, 3, 22]_4$ code with w.d. $0^1 22^{45} 24^{15} 30^3$. Then $\mathcal{C}_3$ is extendable by Thm 7 ($m = 1$, $t = 2$, $p = 2$), for

$$\Sigma_{i \not\equiv d \pmod{2^2}} A_i = 1 + 15 = 2^{3-1}.$$

$\mathcal{C}_3$ is from

I. Bouyukliev, M. Grassl, Z. Varbanov, New bounds for $n_4(k, d)$ and classification of some optimal codes over GF(4), *Discrete Math.*, **281**, 43–66, 2004.

For an $[n,k,d]_q$ code $\mathcal{C}$ with $\gcd(d,q)<q$, the diversity of $\mathcal{C}$ is defined as (Φ_0,Φ_1) with

$$\Phi_0=\frac{1}{q-1}\sum_{q|i,i>0}A_i,\quad \Phi_1=\frac{1}{q-1}\sum_{i\not\equiv 0,d \pmod q}A_i.$$

Note.

Under the condition $\gcd(d,q)=1$,

$\mathcal{C}$ is extendable if $\Phi_1=0$

by Thm 2.

For an $[n,k,d]_q$ code $\mathcal{C}$ with $\gcd(d,q) < q$, the diversity of $\mathcal{C}$ is defined as (Φ_0, Φ_1) with

$$\Phi_0 = \frac{1}{q-1} \sum_{q|i,\, i>0} A_i, \quad \Phi_1 = \frac{1}{q-1} \sum_{i \not\equiv 0, d \pmod q} A_i.$$

Note.

Under the condition $\gcd(d,q) = 1$,

$\mathcal{C}$ is extendable if $\Phi_1 < q^{k-2}$

[M-Yoshida 2012].

Thm 8 (Maruta, 2005).

$\mathcal{C}$: $[n,k,d]_3$ code with diversity (Φ_0,Φ_1),

$\gcd(3,d)=1$, $k \geq 3$.

Then $\mathcal{C}$ is extendable if

$$(\Phi_0,\Phi_1) \in \{(\theta_{k-2},0),\ (\theta_{k-3},2\cdot 3^{k-2}),\ (\theta_{k-2},2\cdot 3^{k-2}),\ (\theta_{k-2}+3^{k-2},3^{k-2})\}$$

where $\theta_j = (3^{j+1}-1)/2$.

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$\mathcal{C}$: $[n, k, d]_3$ code with diversity (Φ_0, Φ_1),

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Then $\mathcal{C}$ is extendable if

$$(\Phi_0, \Phi_1) \in \{(\theta_{k-2}, 0),\ (\theta_{k-3}, 2 \cdot 3^{k-2}),\ (\theta_{k-2}, 2\cdot 3^{k-2}),\ (\theta_{k-2} + 3^{k-2}, 3^{k-2})\}$$

where $\theta_j = (3^{j+1} - 1)/2$.

Thm 9.

$\mathcal{C}$: $[n, k, d]_q$ code with diversity (Φ_0, Φ_1),

$\gcd(d, q) = 1$, $k \geq 3$.

Then $\mathcal{C}$ is extendable if

$$(\Phi_0, \Phi_1) = (\theta_{k-1} - 2q^{k-2}, q^{k-2})$$

where $\theta_j = (q^{j+1} - 1)/(q - 1)$.

Note.

$\theta_{k-1} - 2q^{k-2} = \theta_{k-2} + 3^{k-2}$ for $q = 3$.

Example 5.

$\mathcal{C}_3$: $[15, 3, 11]_4$ code with generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & \bar{\omega} & \bar{\omega} & 1 & \omega & 1 & \bar{\omega} & \omega & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & \omega & 1 & 0 & 1 & 0 & 0 & \bar{\omega} & 1 \end{bmatrix},$$

where $\mathbb{F}_4 = \{0, 1, \omega, \bar{\omega}\}$. The w.d. of $\mathcal{C}_3$ is $0^1 7^3 8^3 9^3 11^9 12^{36} 13^9$ with diversity $(13, 4)$. So, $\mathcal{C}_3$ is extendable by Thm 9.

Thm 9.

$\mathcal{C}$: $[n,k,d]_q$ code with diversity (Φ_0,Φ_1),

$\gcd(d,q)=1$, $k \geq 3$.

Then $\mathcal{C}$ is extendable if

$$(\Phi_0,\Phi_1)=(\theta_{k-1}-2q^{k-2},q^{k-2})$$

where $\theta_j=(q^{j+1}-1)/(q-1)$.

Question.

How about when $\gcd(d,q)>1$?

Thm 9.

$\mathcal{C}$: $[n, k, d]_q$ code with diversity (Φ_0, Φ_1),

$\gcd(d, q) = 1$, $k \geq 3$.

Then $\mathcal{C}$ is extendable if

$$(\Phi_0, \Phi_1) = (\theta_{k-1} - 2q^{k-2}, q^{k-2})$$

where $\theta_j = (q^{j+1} - 1)/(q - 1)$.

Answer.

OK for $k = 3$, $q = 2^h$, $h \geq 3$ if $\gcd(d, q) = 2$.

4. Geometric approach

$\mathcal{C}$: $[n,k,d]_q$ code, $k \geq 3$, $\gcd(d,q) < q$

$G = [g_1^{\mathsf{T}}, \cdots, g_k^{\mathsf{T}}]^{\mathsf{T}}$: a generator matrix of $\mathcal{C}$

$\Sigma :=$PG$(k-1,q)$: the projective space of dimension $k-1$ over $\mathbb{F}_q$

For $P = \mathbf{P}(p_1,\ldots,p_k) \in \Sigma$, the weight of P w.r.t. G, denoted by $w_G(P)$, is defined as

$$w_G(P) = wt(p_1 g_1 + \cdots + p_k g_k).$$

A hyperplane H of Σ is defined by a non-zero vector $h = (h_0, \ldots, h_{k-1}) \in \mathbb{F}_q^k$ as

$$H = \{P = \mathbf{P}(p_0, \ldots, p_{k-1}) \in \Sigma \mid h_0 p_0 + \cdots + h_{k-1} p_{k-1} = 0\}.$$

h is called a defining vector of H.

Let $F_d = \{P \in \Sigma \mid w_G(P) = d\}$.

Lemma 10. $\mathcal{C}$ is extendable $\Leftrightarrow$ there exists a hyperplane H of Σ s.t. $F_d \cap H = \emptyset$. Moreover, $[G, h]$ generates an extension of $\mathcal{C}$, where $h^\top \in \mathbb{F}_q^k$ is a defining vector of H.

Lemma 10. $\mathcal{C}$ is extendable $\Leftrightarrow$ there exists a hyperplane H of Σ s.t. $F_d \cap H = \emptyset$. Moreover, $[G, h]$ generates an extension of $\mathcal{C}$, where $h^\top \in \mathbb{F}_q^k$ is a defining vector of H.

$$F_0 = \{P \in \Sigma \mid w_G(P) \equiv 0 \pmod{q}\}$$
$$F_1 = \{P \in \Sigma \mid w_G(P) \not\equiv 0, d \pmod{q}\}$$
$$F_2 = \{P \in \Sigma \mid w_G(P) \equiv d \pmod{q}\} \supset F_d$$

Note. • $(\Phi_0, \Phi_1) = (|F_0|, |F_1|)$.

• $F_0 \cup F_1$ forms a blocking set w.r.t. lines.

Lemma 11 (M, 2008).

For a line $L = \{P_0, P_1, \cdots, P_q\}$ in Σ,

$$\sum_{i=0}^{q} w_G(P_i) \equiv 0 \pmod{q}.$$

Lemma 12 (Yoshida-M, 2010).

Let K be a set in $\Sigma = \mathrm{PG}(k-1, q)$, $k \geq 3$, $q = 2^h$, $h \geq 3$, s.t. for any line ℓ,

$$|\ell \cap K| \in \{1, q/2 + 1, q + 1\}.$$

Then, K contains a hyperplane of Σ.

Thm 5.

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $q = 2^h$, $h \geq 3$, $\gcd(d, q) = 2$. Then $\mathcal{C}$ is extendable if

$$A_i > 0 \;\Rightarrow\; i \equiv 0 \text{ or } d \pmod{q}.$$

Note.

- $\mathcal{C}$ is 2-weight (mod q).
- Conditon "$\gcd(d, q) = 2$" is assumed.

Proof of Thm 5 (sketch). For $q = 2^h$, $h \geq 3$

$\mathcal{C}$: $[n, k, d]_q$ code with $\gcd(d, q) = 2$ s.t.

$$A_i > 0 \;\Rightarrow\; i \equiv 0 \text{ or } d \pmod{q}. \qquad (1)$$

L: a line in $\Sigma = \mathrm{PG}(k-1, q) = F_0 \cup F_2$.

Assume $|L \cap F_2| = t$. Lemma 11 and (1) imply

$$td \equiv 0 \pmod{q}, \text{ so, } t \equiv 0 \pmod{q/2},$$

for $\gcd(d, q) = 2$. Hence $t = 0, q/2$ or q.

Thus, $|F_0 \cap L| = 1, q/2 + 1$ or $q + 1$, and F_0 contains a hyperplane of Σ by Lemma 12.

Hence $\mathcal{C}$ is extendable by Lemma 10. □

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Thank you for your attention!