On Integer Code Correcting Single Error of Type $(\pm 1, 2)$

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- Introduction to Codes for Flash Memories
- Notations and Definitions
- Construction of Integer Code Correcting Single Error of Type $(\pm 1, 2)$
- Conclusion and Open Problems

Introduction

- Nonvolatile memory is computer memory that maintains stored information without a power supply - cell phones, mp3 players, digital cameras, notebooks
- Flash memory is currently the dominant nonvolatile memory
  - It consumes less power
  - Can be electrically programmed and erased with relative ease
  - Reading and writing are very fast ($\sim$ 100 times faster than disk)

Flash memory consists of cells that store one or more bits by electrical charge of two or more voltage levels. The cells are organized in blocks. Flash memory has two very specific features:

- The voltage of the charge can easily be increased, but can only be decreased by an erasure operation. Only whole blocks can be erased.
- Erasures are very slow. Each block has a limited number of erase cycles it can handle. After 10,000 - 100,000 erasures, the block cannot be reliably be used

The impossibility to decrease the voltage only in a cell results in asymmetry both in reading and writing processes. This leads to two main problems concerning flash memories

- Development of methods for writing in (re-programming) cells with as minimum as possible erasures (WOM codes, floating codes, flash code, rewriting codes, etc.)
- Development of suitable error correcting codes. Errors occur during the process of reading are with limited magnitude and in one dominant direction (the "read voltage level" is less than the actual one)

Example

Track 4 bits with 8 cells having 4 states

3 2 2 0 3 0 3 1	$\rightarrow$	1 0 1 0	Change bit 3
3 2 2 0 3 1 3 1	$\rightarrow$	1 0 0 0	Change bit 2
3 2 3 0 3 1 3 1	$\rightarrow$	1 1 0 0	Change bit 1
3 3 3 0 3 1 3 1	$\rightarrow$	0 1 0 0	Change bit 1
1 0 1 0 0 0 0 0	$\rightarrow$	1 1 0 0	

Correcting asymmetric errors:

- Conventional (symmetric) error correcting codes as BCH, Reed-Solomon, LDPC codes was first used
- Asymmetric error correcting codes were considered by Varshamov and Tenengolz in 1965
- Multilevel flash memories renew interest in codes correcting asymmetric errors. At ACCT'02 Ahlswede et al. introduced a q-ary asymmetric channel
- Codes over ring $\mathbb{Z}_q$ of integers modulo q specially constructed for correcting asymmetric errors have been proposed by Cassuto et al. (2008)
- Codes based on B sets described by Klove et al. (2011)

Notations and Definitions

Definition

Let $\mathbb{Z}_A$ be the ring of integers modulo A. An *integer code* of length n with parity-check matrix $\mathbf{H} \in \mathbb{Z}_A^{m \times n}$, is referred to as a subset of $\mathbb{Z}_A^n$, defined by

$$\mathcal{C}(\mathbf{H}, \mathbf{d}) = \{\mathbf{c} \in \mathbb{Z}_A^n \,|\, \mathbf{c}\mathbf{H}^T = \mathbf{d} \mod A\}$$

where $\mathbf{d} \in \mathbb{Z}_A^m$.

Definition

Let l_j and e_i be positive integers, $j = 1, \ldots, m$, $i = 1, \ldots, s$. The code $\mathcal{C}(\mathbf{H}, d)$ is said to be a *single* $(l_1, l_2, \ldots, l_m, \pm e_1, \pm e_2, \ldots, \pm e_s)$*-error correctable* if it can correct any single error with value l_j, e_i or $-e_t$.

Definition

A single $(l_1, l_2, \ldots, l_m, \pm e_1, \pm e_2, \ldots, \pm e_s)$-error correctable code $\mathcal{C}(\mathbf{H}, d)$ of block length n is called *perfect*, when $A = (2s + l)n + 1$.

Consider the following sets of integers

- $B = B(m) = \{\, 4^k l < m \,|\, k, l, m \in \mathbb{N},\ l$ is odd and $m \geq 6$ is even$\}$
- $B_0 = \{a \in B \mid 3a \equiv 0 \pmod{2m}$, or $\exists\, b \in B : 2a + b \equiv 0 \pmod{2m}\}$
- $B_1 = B \setminus B_0$

Construction of Integer Code Correcting Single Error of Type $(\pm 1, 2)$

Theorem

Let $m \geq 6$ is a given integer and m is even. Let us consider the sets $B(m), B_0$ and B_1. The integer code $\mathcal{C}(\mathbf{H})$ over Z_{2m} with the check matrix

$$\mathbf{H} = (1, 3, 5, 7, \cdots, m-1 \,|\, B_1)$$

is a single $(\pm 1, 2)$ error-correctable.

Remark:

- From the definition of B_0 we have that if $2a + b \equiv 0 \pmod{2m}$, where $a, b \in B(m)$, then $a \in B_0$ and $b \in B_1$. For every such a pair (a, b), we can choose one of the elements to be in B_0 and the other one in B_1 and the above theorem still holds

Example

Let $m = 64$. For the sets B, B_0 and B_1 we have

$$B_m = \{4, 12, 16, 20, 28, 36, 44, 48, 52, 60\}, \qquad B_0 = \emptyset,$$
$$B_1 = \{4, 12, 16, 20, 28, 36, 44, 48, 52, 60\}.$$

So, the integer code $\mathcal{C}(\mathbf{H})$ over Z_{128} with the check matrix

$$H = (1, 3, 5, 7, \cdots, 63, 4, 12, 16, 20, 28, 36, 44, 48, 52, 60)$$

is a single $(\pm 1, 2)$ error-correctable. The code has length $n = 42$ and it is optimal (the exceeding is 1).

Conclusion and Open Problems

- We presented a new construction of integer code correcting single error of type which consists of symmetric and asymmetric part. In many cases the obtained codes are optimal
- How to construct optimal integer codes correcting single/multiple asymmetric errors of different types suitable for flash memories

THANK YOU FOR

YOUR ATTENTION!