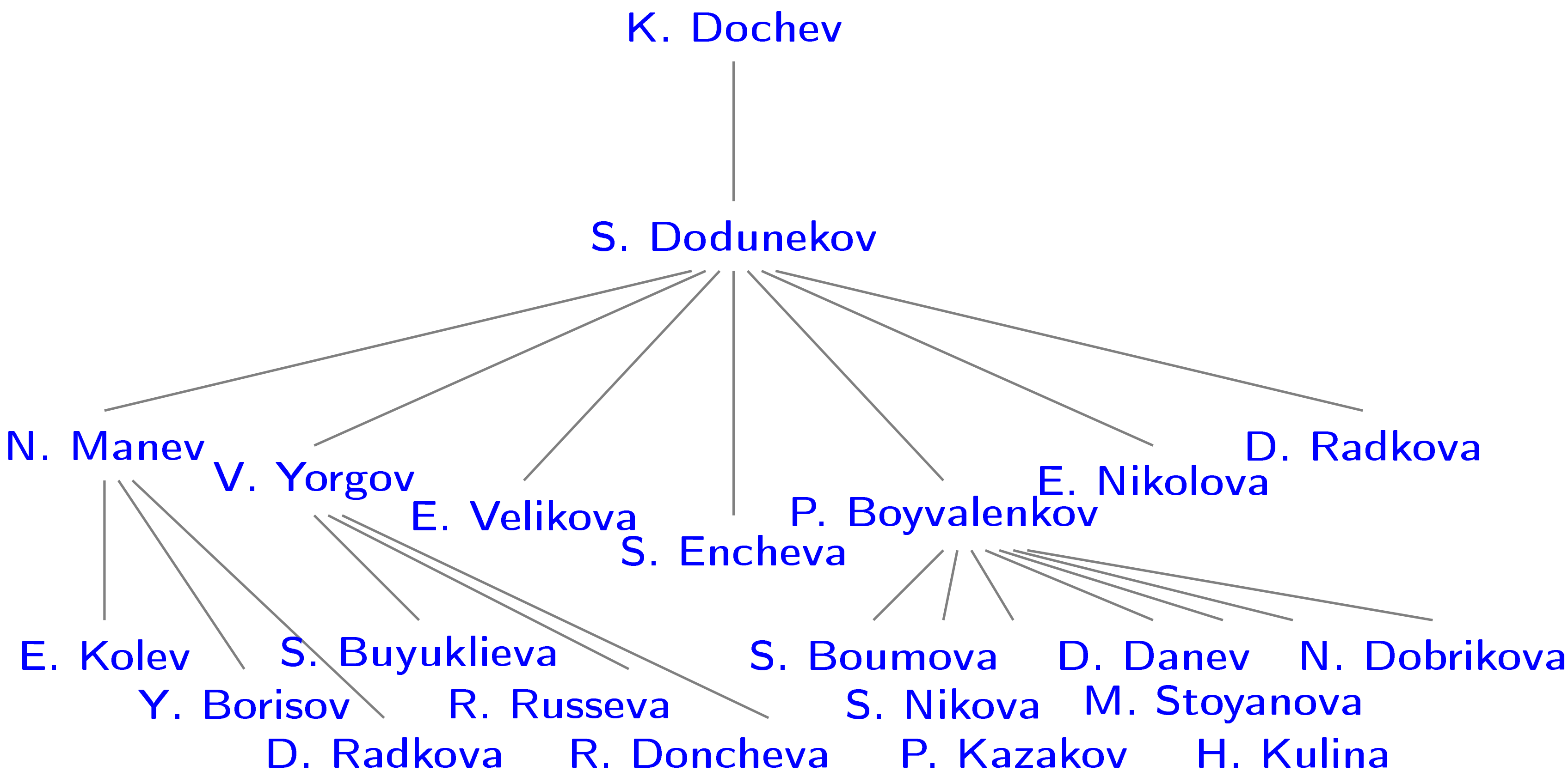
IN MEMORIAM

Stefan Dodunekov (1945–2012)

05.09.1945	born in Kilifarevo, Bulgaria
1963–1968	study in Mathematics at the Faculty of Mathematics and Mechanics, Sofia University
1968	graduates from Sofia University
1969-1972	works at the Regional Computing Center, V. Tarnovo
1972-1981	Assistant Professor at the Faculty of Mathematics and Mechanics, Sofia University
1975	PhD Thesis:: Residual codes and Goppa codes, Supervisor: K. Dochev

1981	Associated Professor, Institute of Mathematics, BAS
1986	Doctor of Sciences, Thesis: Optimal codes
1988	First Workshop on Algebraic and Combinatorial Coding Theory, St. Constantine and Elena
1989	Department of Mathematical Foundation of Informatics
1990	Full Professor, Institute of Mathematics and Informatics, BAS

1995	First Workshop on Optimal Codes and Related Topics
1999-2012	Director of the Institute of Mathematics and Informatics
since 2001	President of the Union of the Bulgarian Mathematicians
2004	Corresponding Member of the Bulgarian Academy of Sciences
2008	Academician
11.06.2012	President of the Bulgarian Academy of Sciences


K. Dochev
S. Dodunekov
N. Manev
V. Yorgov
E. Velikova
S. Encheva
P. Boyvalenkov
E. Nikolova
D. Radkova
E. Kolev
Y. Borisov
D. Radkova
S. Buyuklieva
R. Russeva
R. Doncheva
S. Boumova
S. Nikova
P. Kazakov
D. Danev
M. Stoyanova
H. Kulina
N. Dobrikova

Professor Stefan Dodunekov is the author of over **125 research papers** in algebraic and combinatorial coding theory, **3 books** and **12 textbooks**.

His main scientific achivements are obtained in the following fields:

- theory of the optimal linear codes;
- covering radius of linear codes;
- codes of minimal defect;
- codes in Euclidean spaces;
- decoding algorithms for linear codes.

Optimal Linear Codes

The **Griesmer** bound:

$$n_q(k,d) \geq g_q(k,d) = \sum_{i=0}^{k-1} \lceil \frac{d}{q^i} \rceil$$

For binary codes Belov, Logachev, and Sandimirov gave a necessary and sufficient condition for binary $[g_2(k,d),k,d]$-codes of so-called BV-type to exist.

Independently, Stefan Dodunekov and Ray Hill generalize their result to q-ary codes.

Given q, k and d write d as $d = sq^{k-1} - \sum_{i=1}^{t} q^{u_i - 1}$, where $s = \lceil d/q^{k-1} \rceil$, $k \geq u_1 \geq u_2 \geq \ldots \geq u_t$ and at most $q-1$ u_i's take any given value

Theorem. (S. Dodunekov, R. Hill)

There exists a $[g_q(k,d), k, d]$-code of type BV over the field $\mathbb{F}_q$ if and only if

$$\sum_{i=1}^{\min(s+1,t)} u_i \leq sk.$$

S. Dodunekov, A comment on the weight structure of generator matrices of linear codes, *Problems of Information Transmission*, **26**, No. 2, 1990, 101-104.

Theorem. For every q-ary code of length $n = t + g_q(k, d)$ there exists a generator matrix having as rows only codewords of weight at most $t + d$.

A generalization of the Gray-Rankin bound:

L. A. Bassalygo, S. Dodunekov, V. A. Zinoviev, T. Helleseth, The Grey-Rankin bound for nonbinary codes, *Problems of Information Transmission*, **42**, No. 3, 2006, 37-44.

Theorem. Denote by $S_q(n,d)$ the maximal number of n-simplices in a q-ary Hamming space such that the distance between any two of them is at least d. Then

$$S_q(n,d) \leq \frac{q(qd-(q-2)n)(n-d)}{n-((q-1)n-qd)^2},$$

provided $n-((q-1)n-qd)^2 > 0$. In case of equality the set of all codewords is an orthogonal array of strength 2.

Covering Radius of Linear Codes

- **Conjecture**: For all positive integers n, k with $k \leq n$ and k, there is always a binary $[n, k]$-code of minimal covering radius $R = t(n, k)$ which is self-complementary, i.e. it contains the all-one word.

  G. Cohen, S. Lytsin, I. Honkala, A. Lobstein, *Covering codes*

- A counterexample is constructed in:

  S. Dodunekov, K. Manev, V.Tonchev, On the covering radius of binary (14,6) codes containing the all-one vector, *IEEE Trans. Inform. Theory*, **IT-34**, No. 3, 1988, 591-595.

- Another famous conjecture from F.J. MacWilliams, N. J. A. Sloane, *The Theory of Error-correcting Codes*:

  Conjecture. Apart from the primitive binary BCH codes there are no other quasi-perfect BCH codes.

- D. Danev, S. Dodunekov, A family of ternary quasi-perfect BCH codes, *Designs, Codes and Cryptography*, **49**, No 1, 2008, 265–271.

  Danev and Dodunekov construct a family of ternary quasi-perfect BCH codes.

Codes of Minimal Defect

- S. Dodunekov, I. Landjev, On near-MDS codes, *Journal of Geometry*, **54**, 1995, 30–43.

- S. Dodunekov, I. Landjev, On the quaternary [11,6,5] and [12,6,6] codes, in: "Applications of Finite Fields" (ed. D. Gollmann), IMA Conference Series 59, Clarendon Press, Oxford, 1996, 75–84.

- S. Dodunekov, I. Landjev, Near-MDS Codes over some small fields, *Discrete Mathematics*, **213**, 2000, 55–65.

The Singleton Bound and MDS Codes

$\mathcal{C}$: $[n,k,d]_q$-code

$$\text{SINGLETON}: \quad n \geq d + k - 1,$$

$$\text{SINGLETON}^*: \quad n \geq d_r + k - r, \quad \forall r = 1, \ldots, k.$$

"$=$" in SINGLETON: MDS-codes

"$=$" in SINGLETON* for $r = 1, \ldots, k$: MDS-codes.

MDS $=$ Maximum Distance Separable

The MDS Conjecture

For a non-trivial $[n, k]_q$ MDS code

$$n \leq \begin{cases} q+2 & \text{for } q \text{ even and } k=3 \text{ or } q-1; \\ q+1 & \text{otherwise.} \end{cases}$$

If there exist n points in $\mathrm{PG}(t, q)$ that are in general position then

$$n \leq \begin{cases} q+2 & \text{for } q \text{ even and } t=2 \text{ or } t=q-2; \\ q+1 & \text{otherwise.} \end{cases}$$

For $\mathrm{PG}(t, p)$, p – prime: S. BALL, *J. Europ. Math. Soc.*, 14(2012), 733–748.

Near-MDS Codes

Definition 1. A linear $[n,k]_q$-code $\mathcal{C}$ is called a near-MDS code if $d_i(\mathcal{C}) = n-k+i$, for $i = 2,3,\ldots,k$, and $d_1(\mathcal{C}) = n-k$.

Definition 2. A linear $[n,k]_q$-code $\mathcal{C}$ is called a near-MDS code if there exists a parity check matrix $\mathbf{H}_{\mathcal{C}}$ of $\mathcal{C}$ with the following properties:

(1) any $n-k-1$ columns of $\mathbf{H}_{\mathcal{C}}$ are linearly independent;

(2) there exist $n-k$ linearly dependent columns of $\mathbf{H}_{\mathcal{C}}$;

(3) any $n-k+1$ columns of $\mathbf{H}_{\mathcal{C}}$ have full rank $n-k$.

Definition 3. A linear $[n,k]_q$-code $\mathcal{C}$ is called a near-MDS code if there exists a generator matrix $\mathbf{G}_{\mathcal{C}}$ of $\mathcal{C}$, with the following properties:

(1) any $k-1$ columns of $\mathbf{G}_{\mathcal{C}}$ are linearly independent;

(2) there exist k linearly dependent columns of $\mathbf{G}_{\mathcal{C}}$;

(3) every $k+1$ columns of $\mathbf{G}_{\mathcal{C}}$ have full rank k.

Definition 4. A linear $[n,k]_q$-code $\mathcal{C}$ is called a near-MDS code if

$$d(\mathcal{C}) + d(\mathcal{C}^{\perp}) = n.$$

Results on Near-MDS Codes

Theorem. Let $\mathcal{C}$ be a $[n,k]_q$ near-MDS code. Then for every $s \in \{0,1,\ldots,k\}$:

$$A_{n-k+s} = \binom{n}{k-s}\sum_{j=0}^{s-1}(-1)^j\binom{n-k+s}{j}(q^{s-j}-1) + (-1)^s\binom{k}{s}A_{n-k},$$

Corollary. For an $[n,k]_q$ near-MDS code $\mathcal{C}$

$$A_{n-k} \le \binom{n}{k-1}\frac{q-1}{k},$$

with equality iff $A_{n-k+1} = 0$. By duality

$$A'_k \le \binom{n}{k+1}\frac{q-1}{n-k},$$

with equality iff $A'_{k+1} = 0$.

Problem. Given the positive integer k and the prime power q, find the maximal length $m'(k,q)$ of a near-MDS code of dimension k over $\mathbb{F}_q$.

Theorem. Let k be a positive integer and let q be a prime power. Then

(i) $m'(k,q) \leq 2q + k$;

(ii) $m'(2,q) = 2q + 2$;

(iii) $m'(k,q) \leq m'(k-\alpha,q) + \alpha$, for every α with $0 \leq \alpha \leq k$;

(iv) $m'(k,q) = k+1$, for every $k > 2q$;

(v) $m'(k,q) \leq 2q + k - 2$, for every $q > 3$;

(vi) $m'(2q,q) = 2q + 2$, for every $q > 3$ and every $k > 2$;

(vii) $m'(2q-1,q) = 2q+1$, for every $q > 3$;

$(viii)$ If $q = p^m$,

$$m'(k,q) \geq \begin{cases} q + \lceil 2\sqrt{q} \rceil & \text{if } p \text{ divides } \lceil 2\sqrt{q} \rceil \text{ and } m \geq 3; \\ q + \lceil 2\sqrt{q} \rceil + 1 & \text{otherwise.} \end{cases}$$

Codes in Euclidean Spaces

S. Dodunekov, Th. Ericson, V.A.Zinoviev, Concatenation Methods for Construction of Spherical Codes in N-dimensional Euclidean space, *Problems of Information Transmission*, **27**, No. 4, 1991, 34-38.

Theorem. Let s binary codes $B_i : (w_i, d_{b,i}, M_i)$ and s binary constant weight codes $A_i : (n, w_i, d_i, N_i)$, $i = 1, \ldots, s$, $w_1 < \ldots < w_s$, be given. Then the set of vectors

$$\cup_{i=1}^{s} \cup_{a \in A_i} \mathcal{X}(B_i, a)$$

is a spherical code $Y : (n, \rho, M)$ with

$$\rho = \min(\rho^{(1)}, \rho^{(2)}), \quad M = \sum_{i=1}^{s} M_i N_i,$$

$$\rho^{(1)} = \min_{1 \le i \le s} \left\{ \frac{4d_{b,i}}{w_i}, \frac{d_i}{w_i} \right\},$$

$$\rho^{(2)} = \min_{1 \le i \le s-1} \left\{ 2 - \left(1 - \sqrt{\frac{w_i}{w_{i+1}}} \right) \right\}.$$

In particular, if $d_{i,j} = d(A_i, A_j)$ is the distance between the codes A_i and A_j the above set forms a spherical code $Y : (n, \rho, M)$ with

$$\rho^{(2)} = \min_{i \ne j} \left\{ 2 - \frac{w_i + w_j - d_{i,j}}{\sqrt{w_i w_j}} \right\}.$$

Decoding Algorithms for Linear Codes

- The family of Zetterberg codes with parameters $(2^m + 1, 2^m + 1 - 2u)$ for even m is one of the best known families of double-error correcting codes because of their large code rate.

- The algorithm of Dodunekov-Nillson uses the fact that these codes are quasi-perfect and reversible.

- At the time no procedure for full decoding of the Zetterberg codees was known.

- The ideas were further developed to obtain algorithms for full decoding of the ternary Golay code as well as for the quasi-perfect Gashkov-Sidelnikov codes.

S. Dodunekov, J.Nilsson, Algebraic Decoding of Zetterberg Codes, *IEEE Trans. Inform. Theory*, **IT-38**, No. 5, 1992, 1570-1573.

S. Dodunekov, J. Nilsson, On the decoding of some remarkable ternary codes, *Problems of Information Transmission* **31**, No. 2, 1995, 36-43.

Workshops on Algebraic and Combinatorial Coding Theory

I	1988	St. Constantine and Elena (then Druzhba)
II	1990	Leningrad, Ol'gino
III	1992	Voneshta voda
IV	1994	Novgorod
V	1996	Sozopol
VI	1998	Pskov
VII	2000	Bansko
VIII	2002	Carskoe selo
IX	2004	Kranevo
X	2006	Zvenigorod
XI	2008	Pamporovo
XII	2010	Novosibirsk
XIII	2012	Pomorie

Workshops on Optimal Codes and Related Topics

I	**1995**	Sozopol
II	**1998**	Sozopol
III	**2001**	Sunny Beach
IV	**2005**	Pamporovo
V	**2007**	White Lagoon
VI	**2009**	St. Constantine and Elena
VII	**2013**	Albena