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On the classification of binary self-dual [44, 22, 8] codes with an automorphism of order 3

St. Bouyuklieva
Veliko Tarnovo University, Bulgaria

R. Russeva, N Yankov, N. Ziapkov and M. Nikolova
Shumen University, Bulgaria

$\mathcal{C} - [n, k, d]$ binary linear code

$\mathcal{C}^{\perp}$ is **the dual code** of $\mathcal{C}$ under the standard inner product

$\mathcal{C}$ is a **self-dual** code if $\mathcal{C} = \mathcal{C}^{\perp}$

For SD codes $n = 2k$ and all weights are even

Automorphism of the code $\mathcal{C}$ is a permutation of the coordinates that preserves $\mathcal{C}$

Aut$(\mathcal{C})$ - the group of all automorphisms of $\mathcal{C}$

$\mathcal{C}$ - [44, 22, 8] SD binary code

p=11,7,5,3 - odd primes dividing $|Aut(\mathcal{C})|$

The codes having automorphisms of prime orders $p \geq 5$ are classified

Yorgov(1993),Yorgov and Russeva(1994)
Bouyuklieva(1997,2004), Yankov and Russeva(2008)

We investigate codes with an automorphism of order **3**

$\mathcal{C}$ - [44, 22, 8] SD code with an automorphism σ of type 3-(c,f)

σ - with **c** cycles of order 3 and **f** = 44-3c fixed points in its decomposition

c=6,8,10,12 and 14

c=6 and 14 **(Bouyuklieva(2004), Yankov (2007))**

We classify all codes with an automorphism of order 3 with **8, 10 and 12** 3-cycles

We complete the classification of SD [44,22,8] codes with an automorphism of **odd prime order**

Construction Method

$F_\sigma(\mathcal{C}) = \{v \in \mathcal{C} : v\sigma = v\}$ - the fixed part of $\mathcal{C}$

$E_\sigma(\mathcal{C})$ - the set of vectors in $\mathcal{C}$ with even weight in each 3-cycle of σ and 0 at the fixed points

$$C = F_\sigma(\mathcal{C}) \oplus E_\sigma(\mathcal{C})$$

$$gen(\mathcal{C}) = \begin{pmatrix} gen(F_\sigma(\mathcal{C})) \\ gen(E_\sigma(\mathcal{C})) \end{pmatrix}$$

$\pi : F_\sigma(\mathcal{C}) \longrightarrow F_2^{c+f}$

$\pi(v)$ - vector of length $c+f$ obtained by choosing a coordinate from each cycle and fixed points of v.

$\pi(F_\sigma(\mathcal{C}))$ - a binary self-dual $[c+f, \frac{c+f}{2}]$ code

$E_\sigma(C)^*$ - the code $E_\sigma(C)$ with the last f coordinates deleted.

$\mathcal{P} = \langle x+1 \rangle$ - cyclic code of length 3

$\mathcal{P} \cong F_4$ - set of even weight polynomials

The restriction of $v \in E_\sigma(\mathcal{C})$ on any 3-cycle of σ can be viewed as an element from $\mathcal{P}$

$(v_0, v_1, v_2) \longrightarrow v_0 + v_1 x + v_2 x^2 \in \mathcal{P}$

$$\varphi : \ \mathbf{E}_\sigma(\mathcal{C})^* \rightarrow \mathcal{P}^{\mathbf{c}}.$$

Theorem 1 *A code C with an automorphism σ of order 3 is SD if and only if:*

i) $\pi(F_\sigma(C))$ *is a* $[c+f, \frac{c+f}{2}]$ *binary SD code;*

ii) $\varphi(E_\sigma(C)^*)$ *is a quaternary Hermitian SD code of length* c

Two weight enumerators for [44, 22, 8] SD codes:

$$W_{44,1}(y) = 1 + (44 + 4\beta)y^8 + (976 - 8\beta)y^{10} + \dots$$

for $10 \leq \beta \leq 122$

$$W_{44,2}(y) = 1 + (44 + 4\beta)y^8 + (1232 - 8\beta)y^{10} + \dots$$

for $10 \leq \beta \leq 154$

$W_{44,1}$ and $W_{44,2}$ for various β

Codes with automorphism of type $3-(8,20)$

$\varphi(E_\sigma(C)^*) = C_\varphi$ - unique quaternary [8, 4, 4] code

$\pi(F_\sigma(C)) = C_\pi$ is a $[28, 14, \geq 4]$ SD code

$$G_\pi = \begin{pmatrix} B & O \\ O & D \\ E & F \end{pmatrix}$$

B generates $[8, k_1, \geq 4]$ SO code
D generates $[20, 6 \leq k_1 + 6 \leq 10, \geq 8]$ SO code

All optimal binary self-orthogonal codes of length 20 are classified **(Bouyuklieva(2004))**

23 - [20,6,8] SO codes; 4 - [20,7,8] SO codes; unique [20,8,8] SO code

Hence $k_1 \leq 2$

Theorem 2 *There are exactly **4570** inequivalent [44, 22, 8] SD codes with automorphism of type 3-(8,20).*

Their weight enumerators are of both types $W_{44,1}$ and $W_{44,2}$ with $\beta \leq 76$

Codes with automorphism of type $3-(10,14)$

C_φ - Hermitian [10,5,4] code: E_{10} or B_{10}

C_π is a $[24,12,\geq 4]$ SD binary code

Seven such codes can be used, namely: G_{24}, R_{24}, U_{24}, W_{24}, X_{24}, Y_{24} and Z_{24}

We fix the generator matrix of the subcode C_φ and consider all possibilities for the generator matrix of the subcode C_π

Theorem 3 *There are exactly* ***8738*** *inequivalent [44, 22, 8] SD codes with automorphism of type 3-(10,14).*

1815 codes with $W_{44,1}$ for $\beta \leq 62$

6923 codes with $W_{44,2}$ for $\beta \leq 24$

Codes with automorphism of type $3-(12,8)$

C_φ - Hermitian [12,6,4] code: d_{12}, $2d_6$, $3d_4$, $e_6 \oplus e_6$, and $e_7 + e_5$

C_π is a $[20, 10, \geq 4]$ SD binary code

Seven such codes, namely: $d_{12} + d_8$, $d_{12} + e_8$, d_{20}, d_4^5, $d_6^3 + f_2$, $d_8^2 + d_4$, and $e_7^2 + d_6$

Theorem 4 *There are exactly* ***123147*** *inequivalent [44, 22, 8] SD codes with automorphism of type 3-(12,8).*

Their weight enumerators are of both types

Codes with $W_{44,1}$ for $\beta = 10, \ldots, 68, 70, 72, 74, 82, 86, 90, 122$

Codes with $W_{44,2}$ for $\beta = 0, \ldots, 56, 58, \ldots, 62, 64, 66, 68, 70, 72, 74, 76, 82, 86, 90, 104, 154$

For computing codes - **GAP4.4** and

Q-Extensions (I. Bouyukliev)

THANK YOU!