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Multiple blocking sets in PG(2,23) [1]

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Abstract. An (n, r)-arc is a set of n points of a projective plane such that some r, but no $r + 1$ of them, are collinear. The maximum size of an (n, r)-arc in PG(2, q) is denoted by $m_r(2, q)$. Using some good blocking sets in PG(2, 23) we establish that $m_{22}(2, 23) \geq 484$, $m_{21}(2, 23) \geq 461$, $m_{20}(2, 23) \geq 437$, $m_{19}(2, 23) \geq 411$, $m_{18}(2, 23) \geq 385$ and $m_{17}(2, 23) \geq 360$.

1 Introduction

In this paper we continue our investigations from [3].

Let GF(q) denote the Galois field of q elements and V(3, q) be the vector space of row vectors of length three with entries in GF(q). Let PG(2, q) be the corresponding projective plane. The points of PG(2, q) are the non-zero vectors of V(3, q) with the rule that $X = (x_1, x_2, x_3)$ and $Y = (\lambda x_1, \lambda x_2, \lambda x_3)$ are the same point, where $\lambda \in$ GF(q)$\setminus\{0\}$. The number of points of PG(2, q) is $q^2 + q + 1$. Subspaces of dimension one are called *lines*. The number of lines in PG(2, q) is $q^2 + q + 1$. There are $q + 1$ points on every line and $q + 1$ lines through every point.

Definition 1.1 *An (n, r)-arc is a set of n points of a projective plane such that some r, but no $r + 1$ of them, are collinear.*

Definition 1.2 *An (l, t)-blocking set S in* PG(2, q) *is a set of l points such that every line of* PG(2, q) *intersects S in at least t points, and there is a line intersecting S in exactly t points.*

Note that an (n, r)-arc is the complement of a $(q^2 + q + 1 - n, q + 1 - r)$-blocking set in a projective plane and conversely.

Definition 1.3 *Let M be a set of points in any plane. An i-secant is a line meeting M in exactly i points. Define τ_i as the number of i-secants to a set M.*

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In terms of τ_i the definition of (l,t)-blocking set takes the following form.

Definition 1.4 *An (l,t)-blocking set is a set of l points of a projective plane for which $\tau_i = 0$ for $i < t$, $\tau_t > 0$ and $\tau_i \geq 0$ when $i > t$.*

The next two theorems are proved in [1] and [2] respectively.

Theorem 1.1 *Let B be an (l,t)-blocking set in* PG(2, q) *with $t < q/2$ and $q > 3$ is prime. Then*

$$l \geq (2t+1)(q+1)/2.$$

Theorem 1.2 *Let K be a (k,r)-arc in* PG(2, q) *with $r > (q+3)/2$ and $q \leq 29$ is prime. Then*

$$m_r(2,q) \leq (r-1)q + r - (q+3)/2.$$

2 Blocking sets in PG(2, 23)

The elements of GF(23) will be denoted by $0, 1, 2, \ldots, 9$, $10 = a$, $11 = b$, $12 = c$, $13 = d$, $14 = e$, $15 = f$, $16 = g$, $17 = h$, $18 = i$, $19 = k$, $20 = l$, $21 = m$, $22 = n$.

Theorem 2.1 *There exist a $(69,2)$-blocking set, a $(92,3)$-blocking set, a $(116,4)$-blocking set, a $(142,5)$-blocking set, a $(168,6)$-blocking set and a $(193,7)$-blocking set in* PG(2, 23)*. Therefore,*

$$484 \leq m_{22}(2,23) \leq 492, \quad 461 \leq m_{21}(2,23) \leq 468,$$

$$437 \leq m_{20}(2,23) \leq 444, \quad 411 \leq m_{19}(2,23) \leq 420,$$

$$385 \leq m_{18}(2,23) \leq 396, \quad 360 \leq m_{17}(2,23) \leq 372.$$

Proof:
1. Every three lines in general position in PG(2, 23) forms a $(69,2)$-blocking set. For example, the following set of points

$(0,1,6)$, $(0,1,b)$, $(0,1,h)$, $(1,0,3)$, $(1,0,k)$, $(1,0,l)$, $(1,1,7)$, $(1,1,9)$,
$(1,1,e)$, $(1,2,8)$, $(1,2,f)$, $(1,2,i)$, $(1,3,2)$, $(1,3,6)$, $(1,3,m)$, $(1,4,4)$,
$(1,4,h)$, $(1,4,k)$, $(1,5,5)$, $(1,5,a)$, $(1,5,d)$, $(1,6,7)$, $(1,6,g)$, $(1,7,1)$,
$(1,7,4)$, $(1,7,n)$, $(1,8,5)$, $(1,8,f)$, $(1,8,i)$, $(1,9,3)$, $(1,9,b)$, $(1,9,c)$,
$(1,a,6)$, $(1,a,e)$, $(1,a,h)$, $(1,b,0)$, $(1,b,2)$, $(1,c,6)$, $(1,c,d)$, $(1,c,h)$,
$(1,d,1)$, $(1,d,b)$, $(1,d,c)$, $(1,e,5)$, $(1,e,c)$, $(1,e,i)$, $(1,f,0)$, $(1,f,1)$,
$(1,f,n)$, $(1,g,7)$, $(1,g,b)$, $(1,g,g)$, $(1,h,a)$, $(1,h,d)$, $(1,h,n)$, $(1,i,4)$,
$(1,i,a)$, $(1,i,k)$, $(1,k,m)$, $(1,k,2)$, $(1,l,8)$, $(1,l,9)$, $(1,l,f)$, $(1,m,9)$,
$(1,m,e)$, $(1,m,l)$, $(1,n,3)$, $(1,n,8)$, $(1,n,l)$,

is a $(69, 2)$-blocking set in PG(2, 23) with secant distribution

$$\tau_2 = 66, \quad \tau_3 = 484, \quad \tau_{24} = 3.$$

The complement of this blocking set is a $(484, 22)$-arc in PG(2, 23).

2. The set of points

$(0,1,2),\ (0,1,5),\ (0,1,6),\ (0,1,b),\ (0,1,h),\ (1,0,3),\ (1,0,k),\ (1,0,l),$
$(1,0,m),\ (1,1,3),\ (1,1,7),\ (1,1,9),\ (1,1,e),\ (1,2,8),\ (1,2,f),\ (1,2,i),$
$(1,2,n),\ (1,3,2),\ (1,3,6),\ (1,3,d),\ (1,3,m),\ (1,4,4),\ (1,4,h),\ (1,4,i),$
$(1,4,k),\ (1,5,0),\ (1,5,5),\ (1,5,a),\ (1,5,d),\ (1,6,5),\ (1,6,7),\ (1,6,g),$
$(1,7,1),\ (1,7,4),\ (1,7,a),\ (1,7,n),\ (1,8,5),\ (1,8,f),\ (1,8,i),\ (1,9,3),$
$(1,9,b),\ (1,9,c),\ (1,9,l),\ (1,a,2),\ (1,a,6),\ (1,a,e),\ (1,a,h),\ (1,b,0),$
$(1,b,2),\ (1,b,7),\ (1,c,6),\ (1,c,c),\ (1,c,d),\ (1,c,h),\ (1,d,1),\ (1,d,b),$
$(1,d,c),\ (1,d,h),\ (1,e,5),\ (1,e,c),\ (1,e,i),\ (1,e,n),\ (1,f,0),\ (1,f,1),$
$(1,f,4),\ (1,f,n),\ (1,g,7),\ (1,g,9),\ (1,g,b),\ (1,g,g),\ (1,h,a),\ (1,h,d),$
$(1,h,e),\ (1,h,n),\ (1,i,4),\ (1,i,a),\ (1,i,k),\ (1,k,1),\ (1,k,2),\ (1,k,m),$
$(1,l,6),\ (1,l,8),\ (1,l,9),\ (1,l,f),\ (1,m,9),\ (1,m,b),\ (1,m,e),\ (1,m,l),$
$(1,n,3),\ (1,n,8),\ (1,n,g),\ (1,n,l),$

forms a $(92, 3)$-blocking set in PG(2, 23) with secant distribution

$$\tau_3 = 123, \quad \tau_4 = 388, \quad \tau_5 = 37, \quad \tau_6 = 1, \quad \tau_{24} = 4.$$

The complement of this blocking set is a $(461, 21)$-arc in PG(2, 23).

3. The set of points

$(0,1,6),\ (0,1,b),\ (0,1,c),\ (0,1,h),\ (0,1,m),\ (1,0,3),\ (1,0,4),\ (1,0,f),$
$(1,0,k),\ (1,0,l),\ (1,1,7),\ (1,1,9),\ (1,1,d),\ (1,1,e),\ (1,1,g),\ (1,2,5),$
$(1,2,8),\ (1,2,b),\ (1,2,f),\ (1,2,i),\ (1,3,2),\ (1,3,6),\ (1,3,9),\ (1,3,h),$
$(1,3,m),\ (1,4,4),\ (1,4,6),\ (1,4,7),\ (1,4,h),\ (1,4,k),\ (1,5,5),\ (1,5,a),$
$(1,5,d),\ (1,5,i),\ (1,6,3),\ (1,6,4),\ (1,6,7),\ (1,6,d),\ (1,6,g),\ (1,7,1),$
$(1,7,4),\ (1,7,k),\ (1,7,n),\ (1,8,5),\ (1,8,8),\ (1,8,f),\ (1,8,i),\ (1,8,n),$
$(1,9,3),\ (1,9,b),\ (1,9,c),\ (1,9,l),\ (1,a,0),\ (1,a,6),\ (1,a,9),\ (1,a,e),$
$(1,a,h),\ (1,a,i),\ (1,b,0),\ (1,b,2),\ (1,b,g),\ (1,b,m),\ (1,c,6),\ (1,c,a),$
$(1,c,d),\ (1,c,e),\ (1,c,h),\ (1,c,i),\ (1,d,1),\ (1,d,b),\ (1,d,c),\ (1,d,n),$
$(1,e,5),\ (1,e,a),\ (1,e,b),\ (1,e,c),\ (1,e,i),\ (1,f,0),\ (1,f,1),\ (1,f,8),$
$(1,f,n),\ (1,g,6),\ (1,g,7),\ (1,g,b),\ (1,g,c),\ (1,g,g),\ (1,h,1),\ (1,h,4),$
$(1,h,a),\ (1,h,d),\ (1,h,n),\ (1,i,2),\ (1,i,4),\ (1,i,a),\ (1,i,d),\ (1,i,k),$
$(1,k,0),\ (1,k,2),\ (1,k,a),\ (1,k,m),\ (1,l,4),\ (1,l,8),\ (1,l,9),\ (1,l,e),$
$(1,l,f),\ (1,l,m),\ (1,m,3),\ (1,m,9),\ (1,m,e),\ (1,m,k),\ (1,m,l),\ (1,n,3),$
$(1,n,8),\ (1,n,f),\ (1,n,h),\ (1,n,l),$

forms a $(116, 4)$-blocking set in PG(2, 23) with secant distribution

$$\tau_4 = 178, \quad \tau_5 = 280, \quad \tau_6 = 78, \quad \tau_7 = 12, \quad \tau_{24} = 5.$$

The complement of this blocking set is a $(437, 20)$-arc in PG(2, 23).

4. The set of points

$(0,1,6)$, $(0,1,8)$, $(0,1,c)$, $(0,1,f)$, $(0,1,h)$, $(0,1,m)$, $(0,1,n)$, $(1,0,2)$,
$(1,0,3)$, $(1,0,8)$, $(1,0,h)$, $(1,0,k)$, $(1,0,l)$, $(1,0,m)$, $(1,1,3)$, $(1,1,6)$,
$(1,1,8)$, $(1,1,h)$, $(1,1,l)$, $(1,2,4)$, $(1,2,9)$, $(1,2,d)$, $(1,2,e)$, $(1,2,l)$,
$(1,3,1)$, $(1,3,2)$, $(1,3,8)$, $(1,3,9)$, $(1,3,f)$, $(1,3,m)$, $(1,3,n)$, $(1,4,0)$,
$(1,4,2)$, $(1,4,3)$, $(1,4,7)$, $(1,4,g)$, $(1,4,m)$, $(1,5,4)$, $(1,5,8)$, $(1,5,a)$,
$(1,5,f)$, $(1,5,k)$, $(1,5,m)$, $(1,6,0)$, $(1,6,a)$, $(1,6,d)$, $(1,6,k)$, $(1,6,n)$,
$(1,7,7)$, $(1,7,8)$, $(1,7,b)$, $(1,7,f)$, $(1,7,g)$, $(1,7,h)$, $(1,8,0)$, $(1,8,1)$,
$(1,8,6)$, $(1,8,7)$, $(1,8,f)$, $(1,8,g)$, $(1,8,n)$, $(1,9,0)$, $(1,9,1)$, $(1,9,5)$,
$(1,9,c)$, $(1,9,d)$, $(1,9,i)$, $(1,9,n)$, $(1,a,1)$, $(1,a,9)$, $(1,a,b)$, $(1,a,c)$,
$(1,a,e)$, $(1,b,6)$, $(1,b,9)$, $(1,b,d)$, $(1,b,h)$, $(1,b,i)$, $(1,c,0)$, $(1,c,1)$,
$(1,c,2)$, $(1,c,7)$, $(1,c,m)$, $(1,d,5)$, $(1,d,6)$, $(1,d,a)$, $(1,d,d)$, $(1,d,e)$,
$(1,d,h)$, $(1,e,3)$, $(1,e,5)$, $(1,e,b)$, $(1,e,c)$, $(1,e,i)$, $(1,f,1)$, $(1,f,3)$,
$(1,f,5)$, $(1,f,f)$, $(1,f,i)$, $(1,f,l)$, $(1,g,1)$, $(1,g,4)$, $(1,g,b)$, $(1,g,c)$,
$(1,g,n)$, $(1,h,4)$, $(1,h,7)$, $(1,h,g)$, $(1,h,k)$, $(1,h,l)$, $(1,i,1)$, $(1,i,4)$,
$(1,i,5)$, $(1,i,a)$, $(1,i,d)$, $(1,i,g)$, $(1,i,i)$, $(1,i,k)$, $(1,k,4)$, $(1,k,b)$,
$(1,k,c)$, $(1,k,e)$, $(1,k,g)$, $(1,k,h)$, $(1,k,k)$, $(1,l,2)$, $(1,l,3)$, $(1,l,6)$,
$(1,l,e)$, $(1,l,l)$, $(1,l,m)$, $(1,m,1)$, $(1,m,5)$, $(1,m,8)$, $(1,m,c)$, $(1,m,f)$,
$(1,m,i)$, $(1,n,7)$, $(1,n,9)$, $(1,n,a)$, $(1,n,d)$, $(1,n,e)$,

forms a $(142, 5)$-blocking set in PG(2, 23) with secant distribution

$$\tau_5 = 201, \quad \tau_6 = 210, \quad \tau_7 = 98, \quad \tau_8 = 30, \quad \tau_9 = 7, \quad \tau_{10} = 1 \quad \tau_{24} = 6.$$

The complement of this blocking set is a $(411, 19)$-arc in PG(2, 23).

5. The set of points

$(0,0,1)$, $(0,1,2)$, $(0,1,6)$, $(0,1,8)$, $(0,1,c)$, $(0,1,f)$, $(0,1,h)$, $(0,1,m)$,
$(1,0,2)$, $(1,0,3)$, $(1,0,8)$, $(1,0,f)$, $(1,0,h)$, $(1,0,k)$, $(1,0,l)$, $(1,0,m)$,
$(1,1,3)$, $(1,1,6)$, $(1,1,8)$, $(1,1,h)$, $(1,1,l)$, $(1,2,4)$, $(1,2,9)$, $(1,2,e)$,
$(1,2,h)$, $(1,2,k)$, $(1,2,l)$, $(1,3,1)$, $(1,3,2)$, $(1,3,8)$, $(1,3,9)$, $(1,3,f)$,
$(1,3,g)$, $(1,3,m)$, $(1,3,n)$, $(1,4,0)$, $(1,4,2)$, $(1,4,7)$, $(1,4,e)$, $(1,4,g)$,
$(1,4,m)$, $(1,5,2)$, $(1,5,4)$, $(1,5,8)$, $(1,5,a)$, $(1,5,e)$, $(1,5,f)$, $(1,5,k)$,
$(1,5,m)$, $(1,6,0)$, $(1,6,4)$, $(1,6,a)$, $(1,6,d)$, $(1,6,k)$, $(1,6,n)$, $(1,7,6)$,
$(1,7,7)$, $(1,7,8)$, $(1,7,b)$, $(1,7,f)$, $(1,7,g)$, $(1,7,h)$, $(1,7,l)$, $(1,8,0)$,
$(1,8,1)$, $(1,8,6)$, $(1,8,7)$, $(1,8,8)$, $(1,8,d)$, $(1,8,f)$, $(1,8,g)$, $(1,8,n)$,

$(1,9,1)$, $(1,9,5)$, $(1,9,7)$, $(1,9,a)$, $(1,9,c)$, $(1,9,d)$, $(1,9,i)$, $(1,9,n)$,
$(1,a,1)$, $(1,a,9)$, $(1,a,b)$, $(1,a,c)$, $(1,a,e)$, $(1,b,6)$, $(1,b,9)$, $(1,b,d)$,
$(1,b,e)$, $(1,b,h)$, $(1,b,i)$, $(1,c,0)$, $(1,c,2)$, $(1,c,7)$, $(1,c,g)$, $(1,c,m)$,
$(1,d,5)$, $(1,d,6)$, $(1,d,a)$, $(1,d,d)$, $(1,d,e)$, $(1,d,h)$, $(1,d,i)$, $(1,d,k)$,
$(1,e,3)$, $(1,e,5)$, $(1,e,b)$, $(1,e,c)$, $(1,e,i)$, $(1,e,l)$, $(1,f,1)$, $(1,f,3)$,
$(1,f,5)$, $(1,f,f)$, $(1,f,i)$, $(1,f,l)$, $(1,f,n)$, $(1,g,1)$, $(1,g,4)$, $(1,g,b)$,
$(1,g,c)$, $(1,g,n)$, $(1,h,0)$, $(1,h,3)$, $(1,h,4)$, $(1,h,5)$, $(1,h,7)$, $(1,h,g)$,
$(1,h,k)$, $(1,h,l)$, $(1,i,4)$, $(1,i,5)$, $(1,i,7)$, $(1,i,a)$, $(1,i,d)$, $(1,i,i)$,
$(1,i,k)$, $(1,k,0)$, $(1,k,4)$, $(1,k,7)$, $(1,k,9)$, $(1,k,b)$, $(1,k,c)$, $(1,k,g)$,
$(1,k,h)$, $(1,k,k)$, $(1,l,2)$, $(1,l,3)$, $(1,l,6)$, $(1,l,9)$, $(1,l,e)$, $(1,l,l)$,
$(1,l,m)$, $(1,m,1)$, $(1,m,5)$, $(1,m,8)$, $(1,m,b)$, $(1,m,c)$, $(1,m,e)$, $(1,m,f)$,
$(1,m,i)$, $(1,m,k)$, $(1,n,7)$, $(1,n,9)$, $(1,n,a)$, $(1,n,d)$, $(1,n,e)$, $(1,n,l)$,

forms a $(168,6)$-blocking set in PG(2, 23) with secant distribution

$\tau_6 = 209$, $\tau_7 = 179$, $\tau_8 = 89$, $\tau_9 = 53$, $\tau_{10} = 10$, $\tau_{11} = 4$, $\tau_{12} = 2$, $\tau_{24} = 7$.

The complement of this blocking set is a $(385,18)$-arc in PG(2, 23).

6. The set of points

$(0,1,1)$, $(0,1,2)$, $(0,1,4)$, $(0,1,8)$, $(0,1,k)$, $(0,1,l)$, $(0,1,m)$, $(1,0,0)$,
$(1,0,7)$, $(1,0,8)$, $(1,0,b)$, $(1,0,c)$, $(1,0,f)$, $(1,0,g)$, $(1,0,m)$, $(1,1,6)$,
$(1,1,8)$, $(1,1,a)$, $(1,1,d)$, $(1,1,f)$, $(1,1,g)$, $(1,1,h)$, $(1,2,3)$, $(1,2,4)$,
$(1,2,b)$, $(1,2,c)$, $(1,2,e)$, $(1,2,k)$, $(1,2,l)$, $(1,3,0)$, $(1,3,1)$, $(1,3,2)$,
$(1,3,9)$, $(1,3,e)$, $(1,3,m)$, $(1,3,n)$, $(1,4,0)$, $(1,4,4)$, $(1,4,5)$, $(1,4,7)$,
$(1,4,g)$, $(1,4,i)$, $(1,4,k)$, $(1,5,2)$, $(1,5,5)$, $(1,5,8)$, $(1,5,9)$, $(1,5,e)$,
$(1,5,f)$, $(1,5,i)$, $(1,5,m)$, $(1,6,0)$, $(1,6,2)$, $(1,6,3)$, $(1,6,4)$, $(1,6,5)$,
$(1,6,a)$, $(1,6,b)$, $(1,6,c)$, $(1,6,d)$, $(1,6,k)$, $(1,6,l)$, $(1,6,n)$, $(1,7,1)$,
$(1,7,6)$, $(1,7,7)$, $(1,7,8)$, $(1,7,g)$, $(1,7,h)$, $(1,7,k)$, $(1,7,n)$, $(1,8,1)$,
$(1,8,2)$, $(1,8,3)$, $(1,8,8)$, $(1,8,f)$, $(1,8,g)$, $(1,8,l)$, $(1,8,m)$, $(1,8,n)$,
$(1,9,1)$, $(1,9,2)$, $(1,9,3)$, $(1,9,a)$, $(1,9,d)$, $(1,9,l)$, $(1,9,m)$, $(1,9,n)$,
$(1,a,5)$, $(1,a,6)$, $(1,a,9)$, $(1,a,a)$, $(1,a,b)$, $(1,a,c)$, $(1,a,h)$, $(1,a,i)$,
$(1,b,7)$, $(1,b,9)$, $(1,b,a)$, $(1,b,d)$, $(1,b,e)$, $(1,b,g)$, $(1,b,h)$, $(1,c,2)$,
$(1,c,7)$, $(1,c,9)$, $(1,c,a)$, $(1,c,d)$, $(1,c,e)$, $(1,c,g)$, $(1,d,5)$, $(1,d,6)$,
$(1,d,a)$, $(1,d,b)$, $(1,d,c)$, $(1,d,h)$, $(1,d,i)$, $(1,d,n)$, $(1,e,0)$, $(1,e,1)$,
$(1,e,2)$, $(1,e,3)$, $(1,e,a)$, $(1,e,d)$, $(1,e,i)$, $(1,e,l)$, $(1,e,m)$, $(1,e,n)$,
$(1,f,1)$, $(1,f,2)$, $(1,f,3)$, $(1,f,8)$, $(1,f,f)$, $(1,f,m)$, $(1,f,n)$, $(1,g,1)$,
$(1,g,6)$, $(1,g,7)$, $(1,g,b)$, $(1,g,g)$, $(1,g,h)$, $(1,g,n)$, $(1,h,3)$, $(1,h,4)$,
$(1,h,a)$, $(1,h,b)$, $(1,h,c)$, $(1,h,d)$, $(1,h,k)$, $(1,h,l)$, $(1,i,2)$, $(1,i,4)$,
$(1,i,5)$, $(1,i,8)$, $(1,i,9)$, $(1,i,e)$, $(1,i,f)$, $(1,i,i)$, $(1,i,k)$, $(1,i,m)$,

$(1,k,0)$, $(1,k,4)$, $(1,k,5)$, $(1,k,6)$, $(1,k,7)$, $(1,k,c)$, $(1,k,g)$, $(1,k,i)$, $(1,k,k)$, $(1,l,0)$, $(1,l,1)$, $(1,l,2)$, $(1,l,9)$, $(1,l,e)$, $(1,l,l)$, $(1,l,m)$, $(1,l,n)$, $(1,m,3)$, $(1,m,4)$, $(1,m,5)$, $(1,m,b)$, $(1,m,c)$, $(1,m,k)$, $(1,m,l)$, $(1,n,6)$, $(1,n,7)$, $(1,n,8)$, $(1,n,9)$, $(1,n,a)$, $(1,n,d)$, $(1,n,f)$, $(1,n,g)$, $(1,n,h)$,

forms a $(193,7)$-blocking set in PG(2, 23) with secant distribution

$$\tau_7 = 193, \quad \tau_8 = 187, \quad \tau_9 = 98, \quad \tau_{10} = 46, \quad \tau_{11} = 13,$$

$$\tau_{12} = 6, \quad \tau_{14} = 1, \quad \tau_{22} = 1, \quad \tau_{24} = 8.$$

The complement of this blocking set is a $(360,17)$-arc in PG(2, 23).

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