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On the classification of doubly resolvable designs

SVETLANA TOPALOVA[1] svetlana@moi.math.bas.bg
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
P.O. Box 323, 5000 Veliko Tarnovo, BULGARIA

STELA ZHELEZOVA st_zhelezova@abv.bg
V. Grancharov High School G. Oryahovitsa, BULGARIA

Abstract. Resolvable designs with parallel classes of size q correspond to equidistant codes over Z(q), while doubly resolvable 2-(v,k,λ) designs also correspond to Kirkman squares $KS_k(v;1,\lambda)$. We classify doubly resolvable designs with definite parameters.

1 Introduction

For the basic concepts and notations concerning combinatorial designs refer, for instance, to [1], [3], [15].

Let $V = \{P_i\}_{i=1}^{v}$ be a finite set of *points*, and $\mathcal{B} = \{B_j\}_{j=1}^{b}$ – a finite collection of k-element subsets of V, called *blocks*. We say that $D = (V, \mathcal{B})$ is a *design* with parameters t-(v,k,λ), if any t-subset of V is contained in exactly λ blocks of $\mathcal{B}$.

Two designs are *isomorphic* if there exists a one-to-one correspondence between the point and block sets of the first design and respectively, the point and block sets of the second design, and if this one-to-one correspondence does not change the incidence.

An *automorphism* of the design is a permutation of the points that transforms the blocks into blocks.

One of the most important properties of a design is its resolvability. The design is *resolvable* if it has at least one resolution.

A *resolution* is a partition of the blocks into subsets called *parallel classes* such that each point is in exactly one block of each parallel class. Two resolutions are isomorphic if there exists an automorphism of the design transforming each parallel class of the first resolution into a parallel class of the second one.

There are already quite a lot of works on the existence or classification of resolvable BIBDs with definite parameters, see for instance [2], [7], [8], [12], [13],

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[14]. It is interesting to point out that in some recent works the classification was only possible after using parameter-specific restrictions on the corresponding equidistant codes.

Double resolvability is of particular interest. A 2-(v,k,λ) design is *doubly resolvable* if it has two distinct resolutions such that each pair of parallel classes, one of the first, and the other of the second resolution, have at most one common block.

Papers on doubly resolvable designs mainly deal with the setting of the existence problem [4], [5], [6], [11], while the aim of the present work is the classification of doubly resolvable designs with certain parameters. We approach the problem by directly constructing only designs which are doubly resolvable. For some parameter cases for which all the resolvable designs have already been constructed, we obtain the same result by applying a double resolvability test to all the nonisomorphic resolutions of these designs.

2 Double resolvability test

We modify a standard resolvability test by adding the additional restriction that any parallel class should not contain more than one block of a parallel class of a given resolution of this design. We apply the test to each nonisomorphic resolution of the design.

Let us consider only some of the points of the design and the blocks they are incident with. We experimented how the double resolvability test works on this structure. If the whole design is doubly resolvable, any such structure is obviously doubly resolvable too. If the design is not doubly resolvable, and we take less than two thirds of its points, we usually obtain a doubly resolvable structure.

3 Construction and results

We construct the design resolutions in lexicographic order point by point (To do it faster we actually construct word by word the corresponding equidistant code). After each point we apply a test for equivalence of the partial solution to a previously generated one, and a double resolvability test. The double resolvability test reduces significantly the whole number of solutions, and this makes it possible to classify the doubly resolvable designs with parameters for which the classification of all the nonisomorphic resolutions is a difficult task, and has not been done yet.

The results are presented in Table 1. The first column shows the number of the design in the tables [9], the second its parameters. In the third column we present the number of nonisomorphic resolutions "Nr" known by now, followed by citing of where it is reported (or $*$ if we could not find it calculated by other authors), and the sign $\surd$ if we have independently checked this number by our program too. In the column "drNr" we present the number of nonisomorphic resolutions of the doubly resolvable designs with these parameters, followed by the sign $\surd$ if we have obtained this number in two different ways, namely on the one hand we have directly generated only the doubly resolvable designs, and on the other hand we have checked for double resolvability all the nonisomorphic resolutions of designs with these parameters. We mark the case by $*$ if the second approach cannot be used because of lack of data on all nonisomorphic resolutions "Nr".

Table 1: Doubly resolvable designs

No	BIBD	Nr	drNr	drNd
66	(9,3,3)	426 [10] √	5 √	3
101	(8,4,6)	4 [9] √	1 √	1
145	(9,3,4)	149041 [10] √	83 √	38
235	(9,3,5)	≥203047732 [10]	≥15799 *	≥5941
236	(6,3,8)	1 [9] √	1 √	1
278	(8,4,9)	10 [9] √	1 √	1
319	(12,6,10)	≥400 [9]	1 *	1
524	(8,4,12)	31 [9] √	4 √	4
596	(6,3,12)	1 [9] √	1 √	1
743	(12,6,15)	≥12 [9]	1 *	1
819	(8,4,15)	82 * √	4 √	4
891	(10,5,16)	27121734 [13]	5	5
1078	(6,3,16)	1 [9] √	1 √	1

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