# On Ryabko and Ryabko asymptotically optimal perfect steganographic scheme in a noisy channel 

Valeria Potapova

Institute for Information Transmission Problems
Russian Academy of Science

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## Outline

(1) The Problem of Steganography
(2) Asymptotically Optimal Perfect Steganographic Scheme
(3) The Model of Errors for the Ryabko and Ryabko Scheme

## The Idea of Steganography



## Formal Definition

## Definition

An embedding scheme of quality $T$ is a pair of mappings $E: V \times X \rightarrow S$ and $D: S \rightarrow X$ such that for any message $x \in X$ and any container $v \in V$ the stegoword $s=E(v, x)$ possesses the following properties:
(1) $D(s)=x$
(2) $d(v, s) \leq T$

## Ryabko and Ryabko Scheme

There is a source $\mu$ of containers $v$. Containers are generated as strings of symbols which are i.i.d. random variables from some finite alphabet $\mathbb{A}$. Secret binary messages are independent and generated equiprobably by a source $\omega$. In the channel the warden can intercept and then reads all messages.


## Ryabko and Ryabko Scheme. Construction for the Binary

 CaseThe binary message $x=x_{1} x_{2} x_{3} \ldots$ is embedded into the container $v=v_{1} v_{2} v_{3} v_{4} \ldots, v_{i} \in \mathbb{A}=\{a, b\}$.

- The symbols of $v$ are divided into pairs and renamed in the following way:

$$
a a=u, b b=u, a b=y_{0}, b a=y_{1} .
$$

- The pairs, corresponding to $u$, are idle, but the pairs $y_{i}$ are changed into pairs associated with $y_{x_{1}} y_{x_{2}} y_{x_{3}} \ldots$ in the following way:

$$
\begin{aligned}
\left(s_{2 i-1}, s_{2 i}\right)= & \left(\min \left\{v_{2 i-1}, v_{2 i}\right\}, \max \left\{v_{2 i-1}, v_{2 i}\right\}\right) \text { if the } \\
& \quad \text { embedded } x_{k}=0 \text { and } \\
\left(s_{2 i-1}, s_{2 i}\right)= & \left(\max \left\{v_{2 i-1}, v_{2 i}\right\}, \min \left\{v_{2 i-1}, v_{2 i}\right\}\right) \text { if the } \\
& \text { embedded } x_{k}=1 .
\end{aligned}
$$

## Ryabko and Ryabko Scheme. Construction for the Binary

 Case
## Example

Let the secret message be $x=0110 \ldots$ and the container $v=$ aababaaaabaaaabb.... By renaming pairs we get $v=u y_{1} y_{1} u y_{0} y_{1} u u u \ldots$. We embed $x$ and end up with the stegoword $s=u y_{0} y_{1} u y_{1} y_{0} u u u \ldots=$ aaabbaaabaabaaaabb....

## Single Errors on a Pair of Symbols of the Stegoword

With a single error pairs

- $a a$ and $b b$ turn to pairs $a b$ or $b a$
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## Example

Let the secret message be $x=0110 \ldots$ and the container $v=$ aababaaaabaaaabb.... By renaming pairs we get
$v=u y_{1} y_{1} u y_{0} y_{1} u u u \ldots$. We embed $x$ and end up with the stegoword $s=u y_{0} y_{1} u y_{1} y_{0} u u u \ldots=$ aaabbaaabaabaaaabb....
Assume that two errors have occurred during the transmission and $s^{\prime}=$ baaabaaabaabaaaabb.... The decoding algorithm extracts $x^{\prime}=1110 \ldots$.

## Generalized Scheme for Non-binary Case

Symbols of the container are from the alphabet
$\mathbb{A}=\{0,1,2, \ldots, q-1\}$, which symbols are ordered as integers. The two stages of embedding are the same as for the binary case.

- The symbols of $v$ are divided into pairs and renamed in the following way:

$$
\begin{gathered}
\alpha \alpha=u \text { for all } \alpha \in \mathbb{A} \\
\alpha \beta=y_{0} \text { if } \alpha<\beta \\
\alpha \beta=y_{1} \text { if } \alpha>\beta
\end{gathered}
$$

- The pairs, corresponding to $u$, are idle, but the pairs $y_{i}$ are changed into pairs associated with $y_{x_{1}} y_{x_{2}} y_{x_{3}} \ldots$ in the following way:

$$
\begin{gathered}
\left(s_{2 i-1}, s_{2 i}\right)=\left(\min \left\{v_{2 i-1}, v_{2 i}\right\}, \max \left\{v_{2 i-1}, v_{2 i}\right\}\right) \text { if the } \\
\quad \text { embedded } x_{k}=0 \text { and } \\
\left(s_{2 i-1}, s_{2 i}\right)=\left(\max \left\{v_{2 i-1}, v_{2 i}\right\}, \min \left\{v_{2 i-1}, v_{2 i}\right\}\right) \text { if the } \\
\text { embedded } x_{k}=1 .
\end{gathered}
$$

## The Model of Errors for the Non-binary Case

- With a single error pairs $\alpha \alpha$ turns to $\alpha \beta$ or $\beta \alpha$
- If $\alpha<\beta$, a pair $\alpha \beta$ contains 0 . With the conditional probability $\frac{2}{q-1}$ the pair turns into $\alpha \alpha$. If $\alpha$ turns into $\alpha^{\prime}$ and $\alpha^{\prime}>\beta$ or $\beta$ turns into $\beta^{\prime}$ such that $\alpha>\beta^{\prime}$, the regular reversal happens. The probability of reversal depends on the pair! Say $\alpha$ is $k$-th symbol in the alphabet and $\beta$ is $l$-th symbol $(k<l)$. The conditional probability of the reversal is $\frac{q-1-1}{q-1}+\frac{k}{q-1}$.


## Conclusion

We have investigated the universal perfect steganographic system and its behavior during the transmission via a noisy channel or, the same, a channel with an active warden. If an error in transmitted stegoword happens during the transmission, an insertion/deletion takes place in the embedded secret message.

Thank you for your attention!

