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Better goodness-of-fit statistics for coded FSK decoding. [1]

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Abstract. In previous papers we have considered fast-frequency hopping frequency shift keying. We have proposed a decoder with low error rate in channels with low signal to interference ratio. This decoder uses the Kolmogorov-Smirnov criterion statistics to differentiate transmitted codeword from others. In this paper we consider different goodness-of-fit criterion to improve the decoder error rate. We have discovered that the decoder based on the Barnett-Eison criterion have lower error rates in wideband interference scenario, while the one based on the Anderson criterion is better in case of narrowband interference.

1 Introduction

In this work we consider a very low rate coded modulation based on frequency shift keying designed for channels with a low signal to interference ratio (down to -30 dB). This construction and some of its decoders were earlier described in [6–8].

For a system to work with low SIR in a real channel with unknown state it has to rely on a modulation that doesn't need a known state on the receiver. This include frequency shift keying and differential modulations. In this work we consider FSK for its simplicity.

To increase the resistance to high interference we also use frequency hopping, so that the exact frequencies used in FSK change after each transmission. The easiest way to describe it is by transitting a single unit in a premuted OFDM symbol.

To increase the rate of the coded modulation considered we include several FSK symbols in each OFDM symbol so that no FSK symbols share a subchannel.

In [6–8] a decoder based on a goodness-of-fit criterion was proposed. The Kolmogov-Smirnov criterion was used in [8] and Wilcoxon rank-sum test was used in [6]. In this work we consider two more goodness-of-fit criteria: Anderson [1] and Barnett-Eison [2].

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2 Coded FSK

Let us denote the number of q-FSK symbols in each OFDM symbol by N. Without frequency hopping i-th FSK is transmitted on subchannels $iq+1, \dots, i(q+1)$. In the coded modulation we consider we group i-th FSK symbols in $1, \dots, T$ OFDM symbols into an outer code word. In this work we use $[72, 2, 64]_8$ best linear code as the outer code.

This signal code construction can be also described a frequency division multiplexing with each subband transmitting a Kautz-Singleton code [5]. It was earlier reviewed in [3, 4, 9, 10].

To better handle a strong interference we also introduce a pseudorandom permutation of subchannels in each OFDM symbol. This way the effect of the interference on the each Kautz-Singleton codeword is the same. The receiver inverse the permutation of subchannels before proceeding to the demodulation.

There are several different ways to decode a single Kautz-Singleton codeword. A naive approach is to use energy sum demodulator:

$$D_{energysum}(\mathbf{R}) = \arg\max_i \sum_{t=1}^{T} \mathbf{R}_{it} \tag{1}$$

This demodulator is very simple to implement, but its error correction efficiency is not so good in case of a strong interference. In this work we will only consider goodness-of-fit decoders that are described in the next section.

3 Goodness-of-fit decoders

As the received signal contains a lot of information about interference and only a little about the transmitted signal we need to find a way not to include this harmful information. One such way is to measure distance from each codeword to the received word with a "good" distance metric. Let us describe why a goodness-of-fit criterion statistic is a good candidate for such metric.

Definition 1. *The position of units in each Kautz-Singleton codeword is called a* pattern. *The* correct pattern *is the one corresponding to the transmitted codeword.*

Let us consider the symbols under one pattern as one random variable sample and other symbols as another sample. If the pattern is correct then this two samples have different distributions and if it is not correct they will have the same distribution. Statistics to tell different distributions from the identical ones is known as goodness-of-fit statistics.

Two such statistics were used for decoding in previous works: Kolmogorov-Smirnov statistic in [8] and Wilcoxon sum-of-ranks statistic in [6].

The Anderson criterion defined as:

$$n\omega^2 = \frac{1}{q(q-1)T^3}\left[n\sum_{j=1}^{n}\left(r^i_{(j)} - j\right)^2 + (q-1)T\sum_{j=1}^{(q-1)T}\left(\overline{R}^i_{(j)} - j\right)^2\right] - \text{const}, \tag{2}$$

where $\vec{r^i}$ is the vector of ranks of symbols under i-th pattern in the ordered series of the received word, $\overline{R}^i$ are the ranks of all symbols not under i-th pattern. The lower index $x_{(j)}$ denotes position in the ordered series of x.

Let us now describe the Barnett-Eison statistic. Let us denote the number of symbols under i-th pattern that are located in the j-th quarter of the ordered series of the received word by a^i_j. The statistic is defined as:

$$(a_1 + a_4 - \frac{T}{2})^2 \frac{4(qT-1)}{(q-1)T^2} + \left[(a_4 - a_1)^2 + (a_3 - a_2)^2\right] \frac{2(qT-1)}{(q-1)T^2} \tag{3}$$

4 Simulation Results

To compare the error correction efficiencies of different decoders we have selected the following simulation parameters:

- The length of the OFDM symbols is 2048 (which corresponds to 31 MHz of bandwidth), but only 1663 of subchannels are used for transmission (25 MHz).
- The outer code of the Kautz-Singleton code is $[72, 2, 64]_8$ best linear code.
- The order of FSK modulation is $q = 8$.
- Signal to interference ratio is -30 dB.
- The channel model is COST 207 hilly terrain with relative speed 120 km/h.

Figure 1 displays the error rates for different decoders in a channel without interference. In this scenario the decoder based on the Barnett-Eison statistics (later BE decoder) works as good as the sum-of-ranks decoders and has energy gain of about 1 dB relative to the ones based on Kolmogorov-Smirnov statistic (KS decoder) and Anderson statistic (Anderson decoder).

In scenario with 4MHz interference the Anderson decoder has lowest error rate while the BE decoder has the worst as displayed on figure 2.

Changing the bandwidth of the interference to 8 MHz as seen on figure 3 doesn't change the error rate of the BE decoder very much, while the performance of other decoders deteriorates a lot, especially the rank decoder.

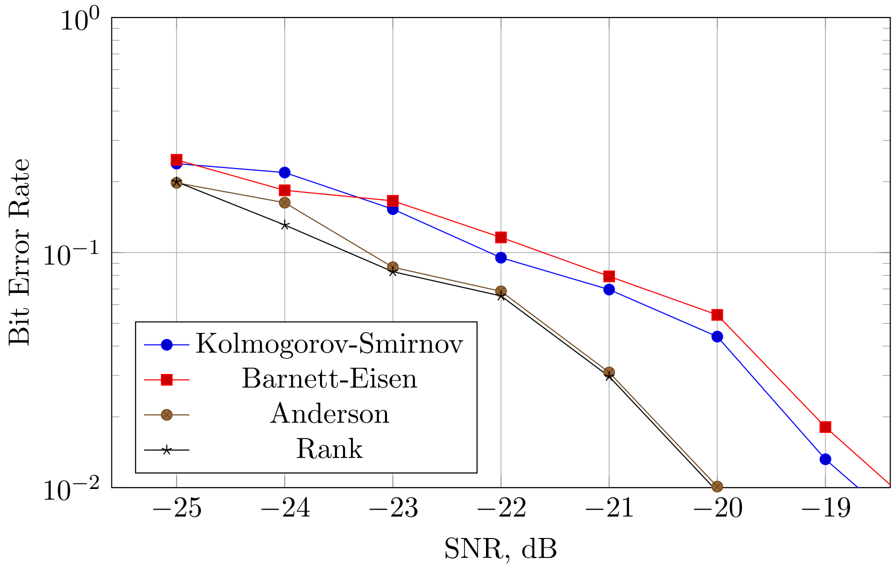

Figure 1: Bit error rate without interference

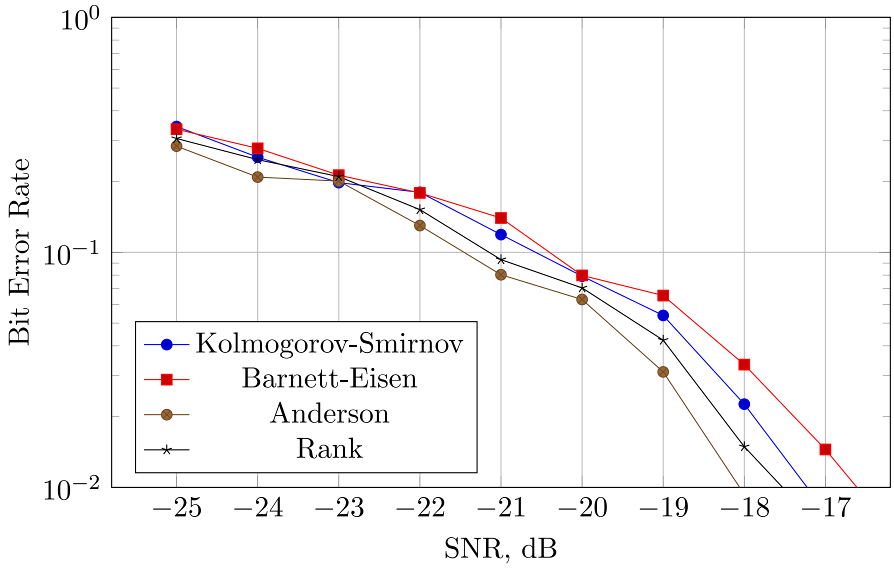

Figure 2: Bit error rate with interference bandwidth 4 MHz

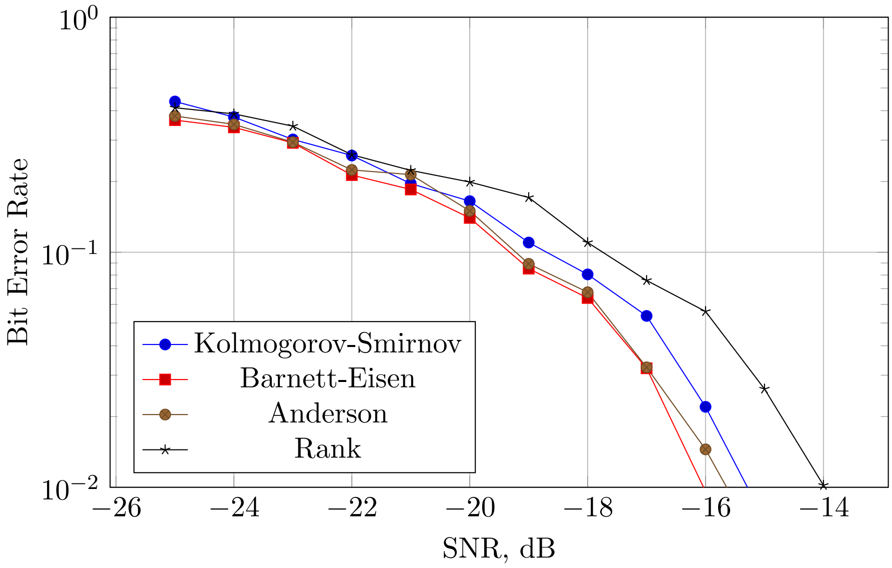

Figure 3: Bit error rate with interference bandwidth 8 MHz

Overall the Anderson decoder perform great for all the above cases. But for 16 MHz interference, displayed on figure 4, it gets an error floor around bit error rate 0.03. The rank decoder doesn't work at all with an interference so wideband. The BE decoder performs the best here with energy gain of about 2 dB relative to the KS decoder.

5 Conclusion

In this work we have considered using different goodness-of-fit statistics for decoding Kautz-Singleton codes with best linear codes as outer codes for channels with strong wideband interference. The previous works have considered using Kolmogorov-Smirnov statistic and the Wilcoxon sum-of-ranks statistic. We have proposed using two statistics: the Barnett-Eison statistic and Anderson statistic.

The computer simulation have shown that Barnett-Eison statistic is the most "interference-proof" among all considered. It has energy gain of 2 dB relative to the Kolmogorov-Smirnov statistic in the worst case and losses 0.25 dB in the channel without interference.

The decoder based on the Anderson statistic has proved to be a good alternative to the rank decoder. It has the same error rate in channels with narrowband interference and has a large advantage in wideband interference channels.

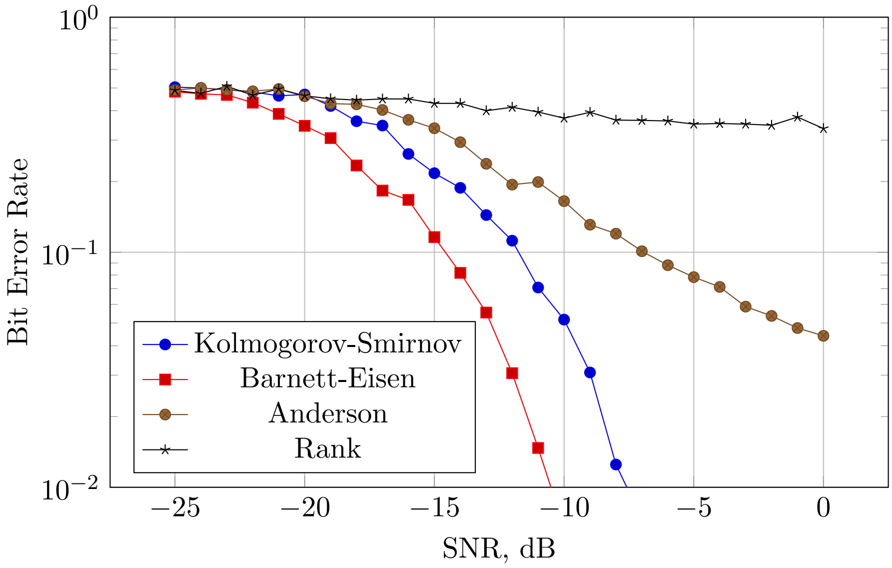

Figure 4: Bit error rate with interference bandwidth 16 MHz

We can conclude that the Barnett-Eison statistic is the best in the worst-case scenario while the Anderson statistic is the best in most scenarios.

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