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A New Error and Erasure Decoding Approach for Cyclic Codes

Alexander Zeh[1,2] and Sergey Bezzateev[3]

[1] Institute of Communications Engineering, Ulm University, Ulm, Germany
[2] INRIA-Saclay-Île de France/École Polytechnique, Paris, France
[3] Saint Petersburg State University of Airspace Instrumentation, St. Petersburg, Russia

June 16, 2012

13th Algebraic and Combinatorial Coding Theory (ACCT 2012), Pomorie, Bulgaria

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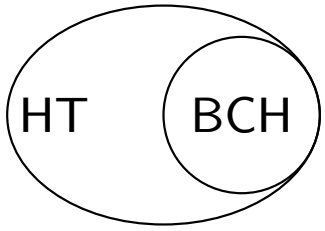

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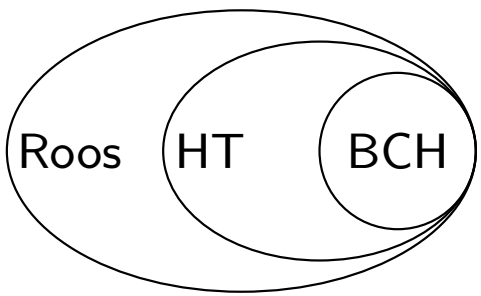

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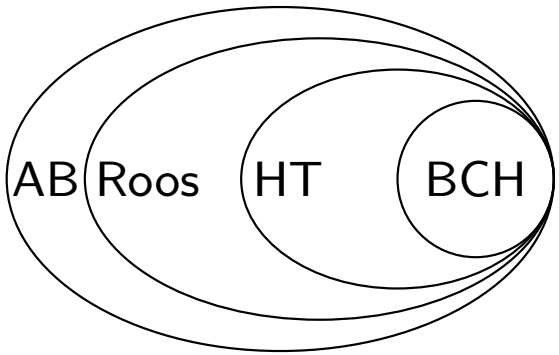

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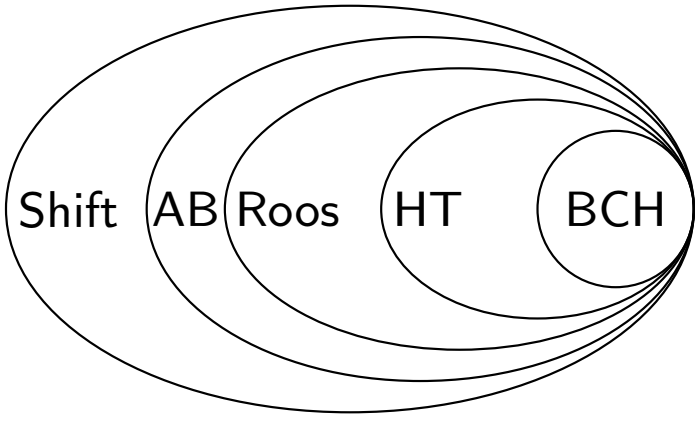

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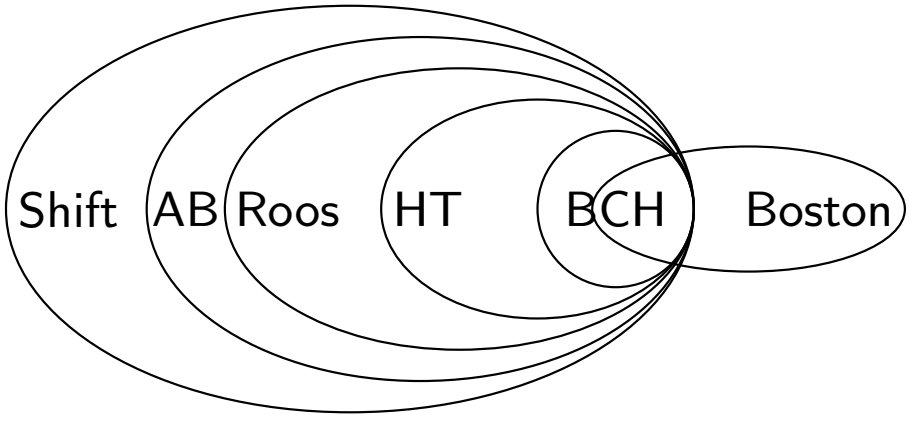

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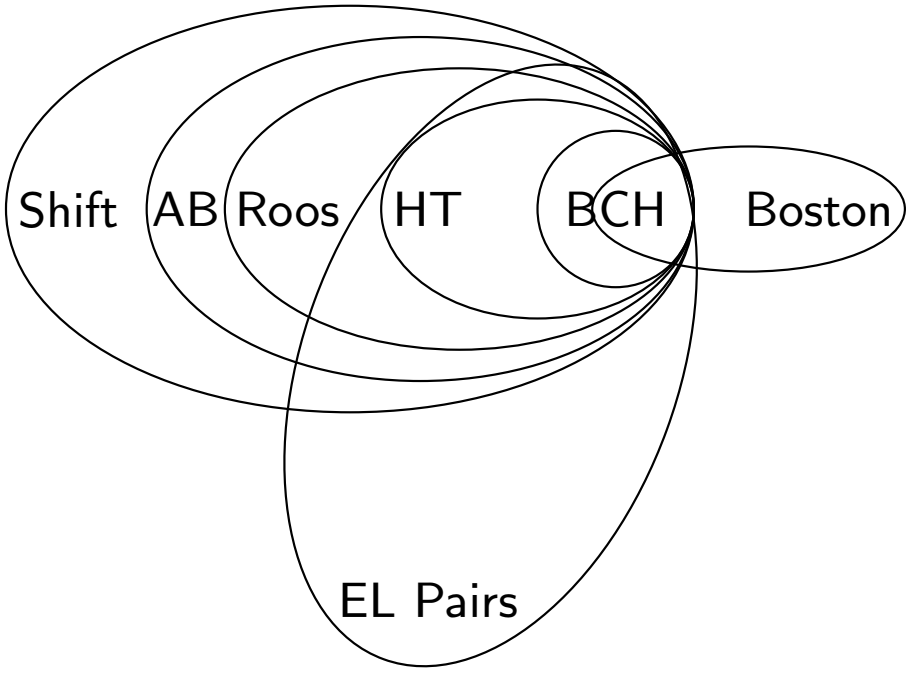

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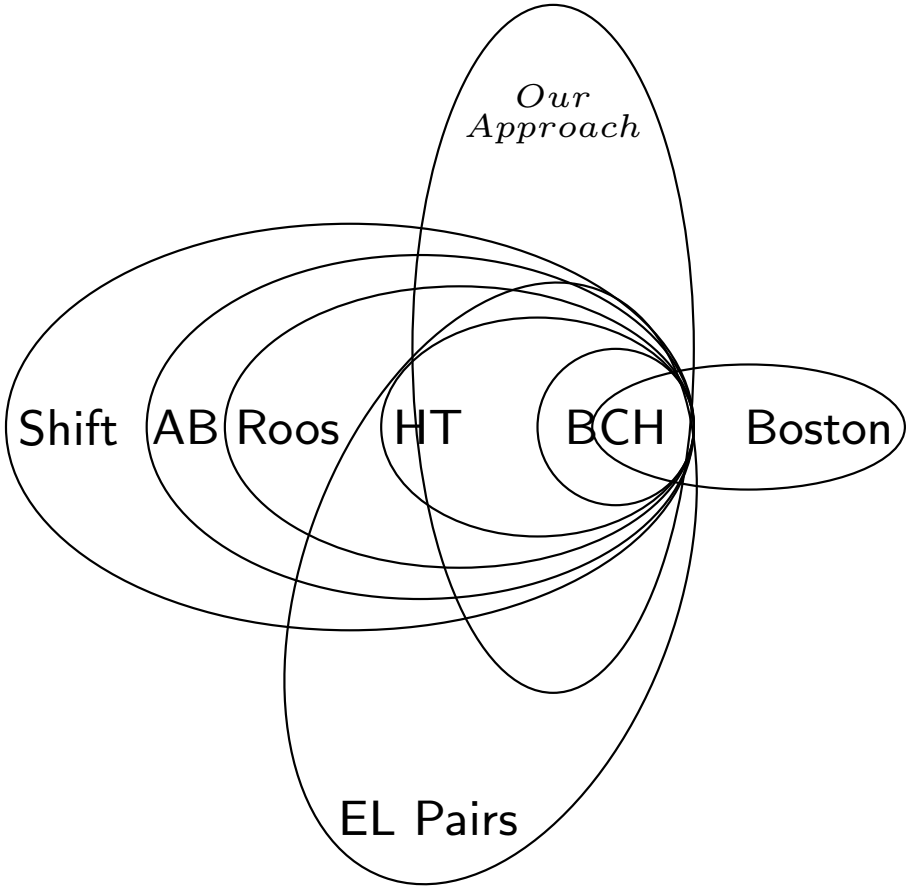

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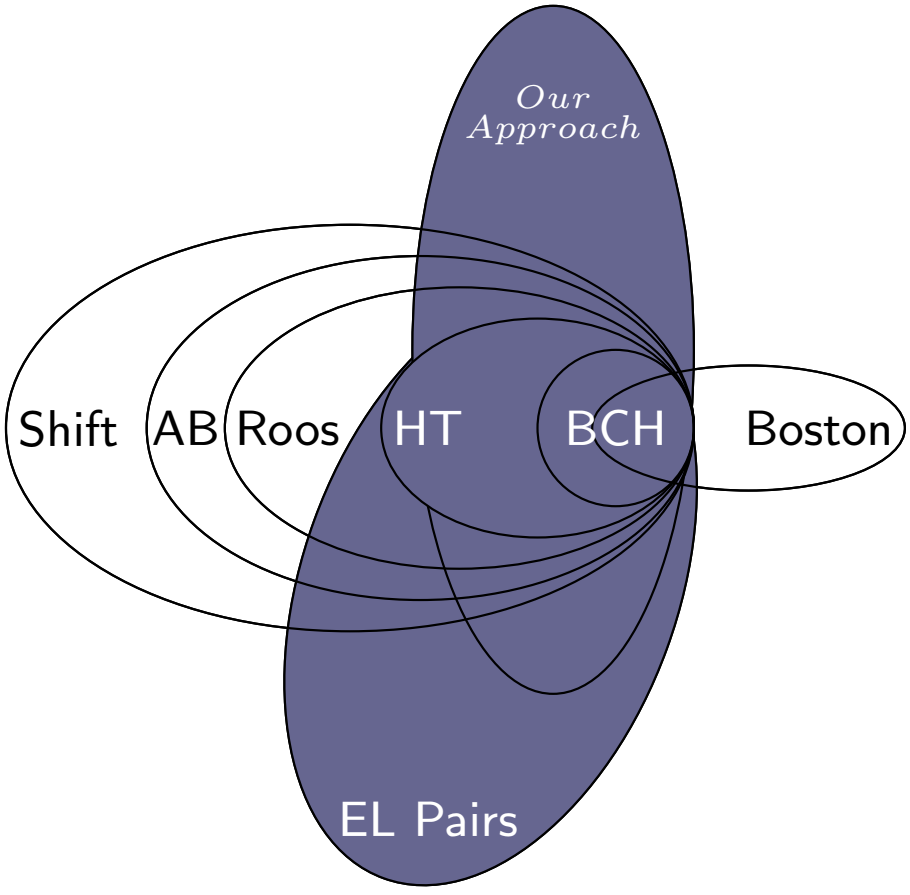

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Outline

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Preliminaries

дефиниция

A q-ary cyclic code over $\mathbb{F}_q$ of

- *length n,*
- *dimension k and*
- *minimum distance d*

is denoted by $\mathcal{C}(q; n, k, d) \subset \mathbb{F}_q^n$.
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The cyclotomic coset $M_r^{(n)}$ modulo n over $\mathbb{F}_q$ is denoted by:

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Defining set $D_{\mathcal{C}}$ of $\mathcal{C}$ is:

$$D_{\mathcal{C}} = \{0 \leq i \leq n-1 \,|\, g(\alpha^i) = 0\} = M_{r_1}^{(n)} \cup M_{r_2}^{(n)} \cup \ldots.$$

Known Bounds on the Minimium Distance

Теорема (Hartmann–Tzeng (HT) Bound)

Given $\mathcal{C}$ with minimum distance d with $D_{\mathcal{C}}$.
Suppose there exist the integers b_1, m_1 and m_2 with $\gcd(n, m_1) = 1$ *and* $\gcd(n, m_2) = 1$ *such that*

$$\{b_1 + i_1 m_1 + i_2 m_2 \mid 0 \leq i_1 \leq d_0 - 2,\ 0 \leq i_2 \leq \nu\} \subseteq D_{\mathcal{C}}.$$

Then $d \geq d_0 + \nu$.

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пример: Binary Cyclic Code of Length 17

The defining set for the binary cyclic code $\mathcal{C}(2; 17, 9, 5)$ of length $n = 17$ is:

$$D_{\mathcal{C}} = M_1^{(17)} = \{1, 2, 4, 8, 9, 13, 15, 16\} \equiv \{1, 2, 4, 8, -8, -4, -2, -1\}.$$

Then, we have $d \geq 5,$ with $b_1 = 1, m_1 = 7, m_2 = 1, d_0 = 3, \nu = 1.$

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$D_{\mathcal{C}}$	13	□	15	16	□	1	2	□	4
$D_{\mathcal{C}}$	-4	□	-2	-1	□	1	2	□	4

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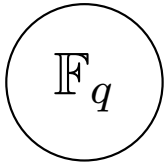
be given.
Then $\mathcal{L}(q_\ell; n_\ell, k_\ell, d_\ell)$ is a non-zero-locator code of $\mathcal{C}$ if there exists a $\mu \geq 2$ and an integer e, such that $\forall\, a(x) \in \mathcal{L}$ and $\forall\, c(x) \in \mathcal{C}$:

$$\sum_{j=0}^{\infty} c(\alpha^{j+e}) a(\beta^j) x^j \equiv 0 \bmod x^{\mu-1}, \tag{1}$$

holds.

The Non-Zero-Locator Code (ii)



Let $r = \mathrm{lcm}(s, u \cdot s_\ell)$ and let γ be a primitive element in $\mathbb{F}_{q^r}$. Then $\gamma^{(q^r-1)/n}$ and $\gamma^{(q^r-1)/n_\ell}$ are elements of order n and n_ℓ.

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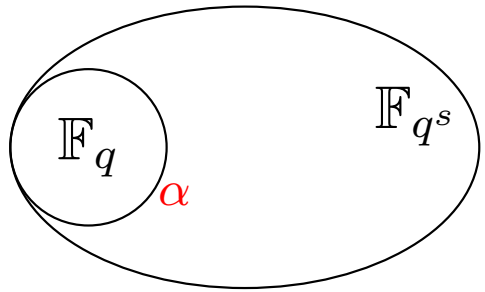

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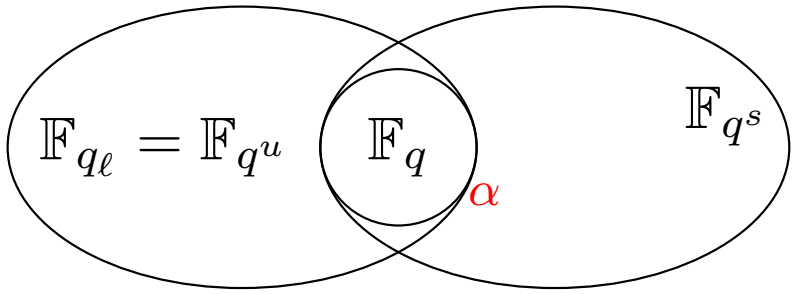

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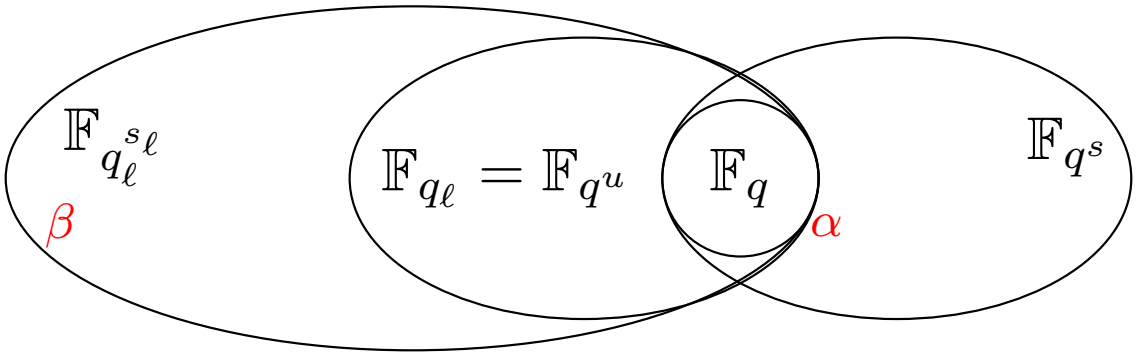

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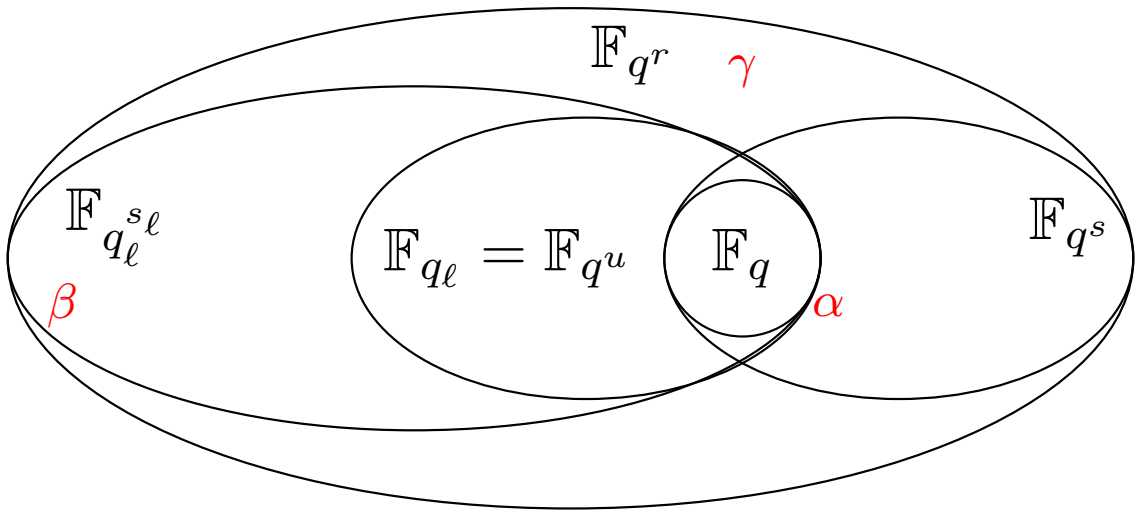

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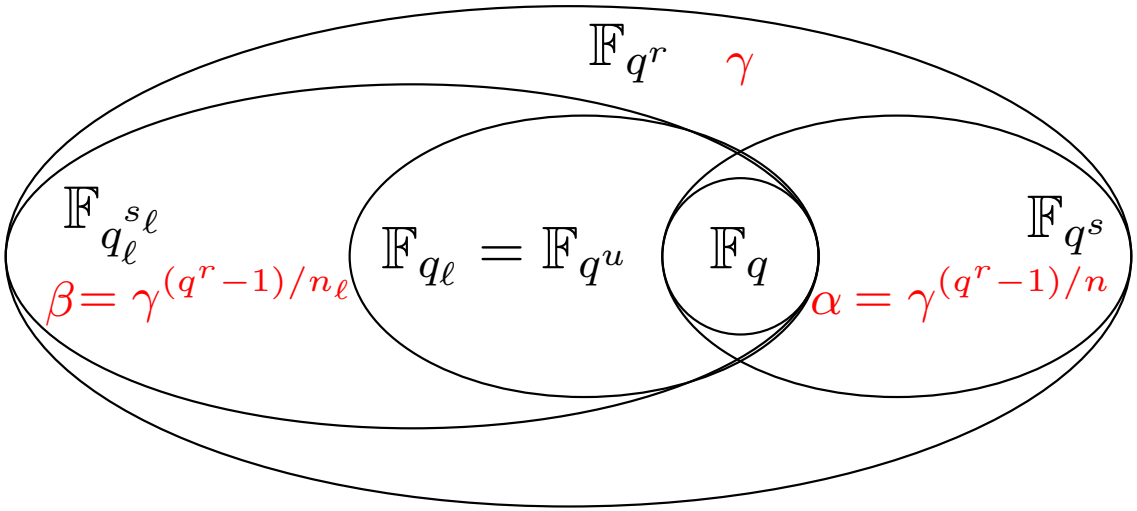

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The Non-Zero-Locator Code (iii)

We rewrite (1).
We search the "longest" sequence:

$$c(\alpha^e)a(\beta^0), c(\alpha^{e+1})a(\beta^1), \ldots, c(\alpha^{e+\mu-2})a(\beta^{\mu-2}),$$

that results in a zero-sequence of length $\mu - 1$.

пример: Binary Cyclic Code of Length 17

$D_{\mathcal{C}}$	-4	□	-2	-1	□	1	2	□	4
$D_{\mathcal{P}}$	□	3	□	□	3	□	□	3	□

Theorem on the Minimum Distance

теорема (Minimum Distance)

Let

- *q-ary cyclic code $\mathcal{C}(q; n, k, d)$ and*
- *non-zero-locator code $\mathcal{L}(q_\ell; n_\ell, k_\ell, d_\ell)$*
- *with $\gcd(n, n_\ell) = 1$ and the integer μ be given.*

Then:

$$d \geq d^* \stackrel{\text{def}}{=} \left\lceil \frac{\mu}{d_\ell} \right\rceil .$$

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Let $a(x) = 1$ be the low-weight codeword $\in \mathcal{L}(q_\ell; n_\ell, n_\ell, 1)$. Then,

$$\sum_{j=0}^{\infty} c(\alpha^{j+e}) a(\beta^j) x^j \equiv 0 \bmod x^{\mu-1},$$

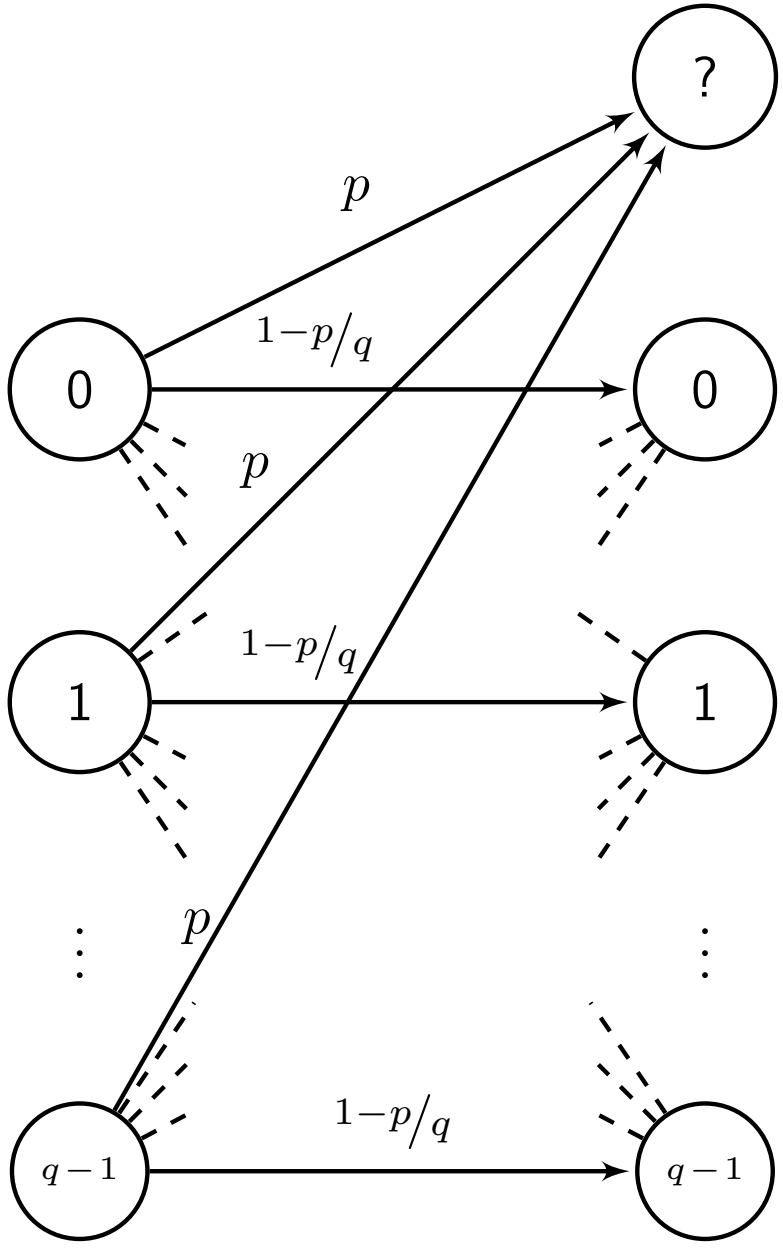
becomes

$$c(\alpha^{j+e}) 1 = 0, \quad \forall j = 0, \ldots, \mu - 1.$$

$\Longrightarrow$ BCH Bound!

Outline

Erasure Channel



Channel Model

- Input: q-ary alphabet
- Output: $(q+1)$-ary alphabet

Erasure probability is p.

Erasures and Received Word

Let:

- the set $\mathcal{E} = \{i_0, i_1, \ldots, i_{\varepsilon-1}\}$ with $|\mathcal{E}| = \varepsilon$ be the set of erroneous positions and
- the polynomial $e(x) = \sum_{i \in \mathcal{E}} e_i x^i$ and
- "?" mark an erasure and
- set $\mathcal{D} = \{j_0, j_1, \ldots, j_{\delta-1}\}$ with $|\mathcal{D}| = \delta$ be the set of erased positions

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and we have the received polynomial:

$$\widetilde{r}(x) = \sum_{i=0}^{n-1} \widetilde{r}_i x^i \quad \text{with} \quad \widetilde{r}_i \in \mathbb{F}_q \cup \{?\}.$$

Erasure Polynomial and Modified Received Word

Substitution of ? by an arbitrary element from $\mathbb{F}_q$ (zero).
We define the erasure polynomial:

$$d(x) = \sum_{i \in \mathcal{D}} d_i x^i,$$

such that $\widetilde{r}_i + d_i = c_i + d_i = 0, \ \forall i \in \mathcal{D}$.

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Let the modified received polynomial $r(x) \in \mathbb{F}_q[x]$ be

$$r(x) = \sum_{i=0}^{n-1} r_i x^i = c(x) + d(x) + e(x).$$

Syndromes and Erasure-Locator Polynomial

Let a low-weight codeword:

$$a(x) = \prod_{j \in \mathcal{Z}} a_j x^j,$$

with $|\mathcal{Z}| = d_\ell$ be given.
We define a syndrome polynomial $S(x) \in \mathbb{F}_{q^r}[x]$ as follows:

$$S(x) \stackrel{\text{def}}{\equiv} \sum_{j=0}^{\infty} r(\alpha^{j+e}) a(\beta^j) x^j \mod x^{\mu-1}.$$

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The corresponding erasure-locator polynomial $\Psi(x) \in \mathbb{F}_{q^r}[x]$ and error-locator polynomial $\Lambda(x) \in \mathbb{F}_{q^r}[x]$ are:

$$\Psi(x) \stackrel{\text{def}}{=} \prod_{i \in \mathcal{D}} \Big(\prod_{j \in \mathcal{Z}} \big(1 - x\alpha^i \beta^j\big) \Big)$$

$$\Lambda(x) \stackrel{\text{def}}{=} \prod_{i \in \mathcal{E}} \Big(\prod_{j \in \mathcal{Z}} \big(1 - x\alpha^i \beta^j\big) \Big).$$

Key Equation

With

$$\widetilde{S}(x) \stackrel{\text{def}}{\equiv} \Psi(x) \cdot S(x) \mod x^{\mu-1}$$

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$$\widetilde{S}(x) \equiv \frac{\widetilde{\Omega}(x)}{\Lambda(x)} \mod x^{\mu-1}$$

with

$$\begin{aligned} \deg \Lambda(x) &= \varepsilon \cdot d_\ell \\ \deg \widetilde{\Omega}(x) &\leq (\varepsilon + \delta) \cdot d_\ell - 1. \end{aligned}$$

Decoding up to the New Bound

Error/Erasure Decoding

Input: Received Word $\widetilde{r}(x)$, Low-weight Codeword $a(x) = \sum_{j \in \mathcal{Z}} a_j x^j \in \mathcal{L}$, Integers e and μ

Prepr.: Calculate one root γ_i of each $\prod_{j \in \mathcal{Z}} \left(1 - x\alpha^i \beta^j\right)$ with $\gamma_i = \beta^{-\kappa} \alpha^{-i}$, where $\kappa \in \mathcal{Z}$

1. Remove erasures from $\widetilde{r}(x) \Rightarrow r(x)$
2. Calculate erasure-polynomial $\Psi(x)$
3. Calculate $S(x)$ and $\widetilde{S}(x)$
4. Obtain $\Lambda(x)$, $\widetilde{\Omega}(x)$ from $\texttt{EEA}\left(x^{\mu-1}, \widetilde{S}(x)\right)$
5. Find all i for which $\Lambda(\gamma_i) = 0$
6. Save them as $\widehat{\mathcal{E}} = \{i_0, i_1, \ldots, i_{\varepsilon-1}\}$ and $\widehat{\mathcal{D}} = \{i_0, i_1, \ldots, i_{\delta-1}\}$
7. Determine Error/Erasure values $e_{i_0}, e_{i_1}, \ldots, e_{i_{\varepsilon-1}}$ and $d_{i_0}, d_{i_1}, \ldots, d_{i_{\delta-1}}$
8. $\widehat{e}(x) \leftarrow \sum_{i \in \widehat{\mathcal{E}}} e_i x^i$ and $\widehat{d}(x) \leftarrow \sum_{i \in \widehat{\mathcal{D}}} d_i x^i$
9. $\widehat{c}(x) \leftarrow r(x) - \widehat{e}(x) - \widehat{d}(x)$

Output: Estimated codeword $\widehat{c}(x)$

Outline

Conclusion and Outlook

Results

The concept of the non-zero-locator code was extended to combined error/erasure decoding:

- A modified **Key Equation** was derived and
- The EEA can be used to solve it.

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- A modified **Key Equation** was derived and
- The EEA can be used to solve it.

Outlook

- Further investigation of "good" non-zero-locator codes.
- Adaption to non-cyclic block codes.
- Sub-quadratic-time modification of the EEA.

Thank you

Благодаря ви за вниманието!