

Example

$\mathbb{F}_9 = \mathbb{F}_3[\alpha]$ with $\alpha^2 + \alpha + 2 = 0$,

- $E = \text{div}(x_0^e) = e \cdot \text{div}(x_0)$, $\deg(E) = e$.
- Curves: $m = 7$ and $d = 1$

$$C_1: x_1 = 0, \quad C_2: x_2 = 0,$$

$$C_3: x_0 + x_1 + x_2 = 0, \quad C_4: \alpha x_0 + x_1 + 2x_2 = 0$$

$$C_5: x_0 + \alpha x_1 + 2x_2 = 0,$$

$$C_6: \alpha^3 x_0 + x_1 + 2x_2 = 0,$$

$$C_7: \alpha^5 x_0 + \alpha^7 x_1 + 2x_2 = 0.$$

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- Points on each curves: $P_{ij} = (1, a_{ij}, b_{ij})$
 $i = 1, \dots, 7, j = 1, \dots, 9$, i.e., $n = 9$.

Remark) $N = \#C_i(\mathbb{F}_9) = 10$ ($\forall i$)

If $0 \leq e \leq 6$, then $n > d \cdot e$ and $m \cdot d > e$,
and $\text{Supp}(E) \cap P_{ij} = \emptyset$.

The code $C(\mathbb{P}^2, \mathcal{C}, E)$ is

$[93, \frac{1}{2}(e+1)(e+2), \delta]$ -code

with $\delta \geq (m - [e/d])(n - de) = (7 - e)(9 - e)$.