A family of binary completely transitive codes and distance-transitive graphs

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Outline

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In particular, for any integer $\rho \geq 2$, there exist two codes in the constructed class of codes with $d = 3$, covering radius ρ and length $\binom{4\,\rho}{2}$ and $\binom{4\,\rho+2}{2}$, respectively.
These new completely transitive codes induce as coset graphs a family of distance-transitive graphs of growing diameter.

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Given any vector $\mathbf{v} \in \mathbb{F}_2^n$ its *distance* to the code C is

$$d(\mathbf{v}, C) = \min_{\mathbf{x} \in C} \{d(\mathbf{v}, \mathbf{x})\}$$

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For a given code C with covering radius $\rho = \rho(C)$ define

$$C(i) \ = \ \{\mathbf{x} \in \mathbb{F}_2^n : \ d(\mathbf{x}, C) = i\}, \ \ i = 1, 2, ..., \rho.$$

Definition 1.

(Neumaier, [1992]) A code C with covering radius $\rho = \rho(C)$ is completely regular, if for all $l \geq 0$ and for every vector $x \in C(l)$ there are precisely:
the same number c_l of neighbors in $C(l-1)$
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Define $a_l = n - b_l - c_l$ and note that $c_0 = b_\rho = 0$.
Define the intersection array of C as $(b_0, \ldots, b_{\rho-1}; c_1, \ldots, c_\rho)$.

For a given code C with automorphism group $\mathrm{Aut}(C)$ and any $\mathbf{x} \in \mathbb{F}_2^n$ and $\varphi \in \mathrm{Aut}(C)$ the group acts on a coset $\mathbf{x} + C$ as

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Definition 2.

(Solé, [1990]) A linear code C with covering radius $\rho=\rho(C)$ and automorphism group $\mathrm{Aut}(C)$ is completely transitive, if the set of all cosets of C is partitioned into $\rho+1$ orbits under action of $\mathrm{Aut}(C)$.

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An *automorphism* of a graph Γ is a permutation π of the vertex set of Γ such that, for all $\gamma, \delta \in \Gamma$ we have $d(\gamma, \delta) = 1$, if and only if $d(\pi\gamma, \pi\delta) = 1$.

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Preliminary results

Definition 3.

(Brouwer-Cohen-Neumaier [1989]) A simple connected graph Γ is called *distance-regular*, if it is regular of valency k, and if for any two vertices $\gamma, \delta \in \Gamma$ at distance i apart, there are precisely:
c_i neighbors of δ in $\Gamma_{i-1}(\gamma)$
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Furthermore, this graph is called *distance transitive*, if for any pair of vertices γ, δ at distance $d(\gamma, \delta)$ there is an automorphism $\pi \in \mathrm{Aut}(\Gamma)$ which moves this pair to any other given pair γ', δ' of vertices at the same distance $d(\gamma, \delta) = d(\gamma', \delta')$.

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Our purpose here is to describe the resulting linear completely transitive codes with growing covering radius and distance-transitive coset graphs with growing diameter.

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Define the graph Γ_C (which is called the *coset graph of* C, taking all cosets $D = C + \mathbf{x}$ as vertices, with two vertices $\gamma = \gamma(D)$ and $\gamma' = \gamma(D')$ adjacent, if and only if the cosets D and D' contains neighbor vertices, i.e. $\mathbf{v} \in D$ and $\mathbf{v}' \in D'$ with distance $d(\mathbf{v}, \mathbf{v}') = 1$.

Lemma 4.

(Brouwer-Cohen-Neumaier [1989], Rifà-Pujol, [1991]) Let C *be a linear completely regular code with covering radius* ρ *and intersection array* $(b_0, \ldots, b_{\rho-1}; c_1, \ldots c_\rho)$ *and let* Γ_C *be the coset graph of* C*.*

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Then Γ_C is distance-regular of diameter ρ with the same intersection array.
If C is completely transitive, then Γ_C is distance-transitive.

Lemma 5.

(Neumaier [1992]) Let C be a completely regular code with covering radius ρ and intersection array $(b_0, \ldots, b_{\rho-1}; c_1, \ldots c_\rho)$. Then $C(\rho)$ is a completely regular code too, with intersection array $(c_\rho, \ldots, c_1; b_{\rho-1}, \ldots b_0)$.

Main results

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Definition 6.

Let H_m be the binary matrix of size $m \times m(m-1)/2$, whose columns are exactly all different vectors of length m and weight 2. Now define the binary linear code $C^{(m)}$ whose parity check matrix is the matrix H_m.

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Code $C^{(m)}$ is completely transitive and, therefore, completely regular. The intersection numbers of $C^{(m)}$ for $i = 0, \ldots, \rho$ are:
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$b_i = \binom{m-2i}{2}, \quad c_i = \binom{2i}{2}.$
Code $C^{(m)}$ is antipodal if m is odd and non-antipodal if m is even.

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The generating matrix $G^{[m]}$ of this code has a very symmetric structure:

$$G^{[m]} \;=\; \left[\begin{array}{c|c} I_{k-1} & H^t_{m-1} \\ \hline 0\dots 0 & 1\dots 1 \end{array}\right].$$

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Using Lemma 5 and the fact that

$$C^{(m)}(\rho) = C^{(m)} + (1,1,\ldots,1),$$

we obtain the following result.

Theorem 8.

Let $m \geq 6$ be even. The code $C^{[m]}$ is completely transitive $[n, k, d]$ code with parameters
$n = m(m-1)/2, \; k = n - m + 2, \; d = 3, \; \rho = \lfloor m/4 \rfloor$.

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$n = m(m-1)/2, \; k = n - m + 2, \; d = 3, \; \rho = \lfloor m/4 \rfloor.$
The intersection numbers of $C^{[m]}$ *for* $m \equiv 0 \pmod 4$ *and* $\rho = m/4$ *are* $b_i = \binom{m-2i}{2} \; c_i = \binom{2i}{2}, \; i = 0, 1, \ldots, \rho - 1,$
$c_\rho = 2\binom{2\rho}{2}$
and, for $m \equiv 2 \pmod 4$ *and* $\rho = (m-2)/4$*, are* $b_i = \binom{m-2i}{2}$,
$c_i = \binom{2i}{2}$, $i = 0, 1, \ldots, \rho$.

We note that the extension of the code $C^{[m]}$ (i.e. adding one more overall parity checking position) is not uniformly packed in the wide sense, and therefore, it is not completely regular (Brouwer et alt. [1989]).

Denote by $\Gamma^{(m)}$ (respectively, $\Gamma^{[m]}$) the coset graph, obtained from the codes $C^{(m)}$ (respectively, $C^{[m]}$) by Lemma 4. From Theorems 7 and 8 we obtain the following results, which leads to new coset graphs.

Theorem 9.

For any even $m \geq 6$ *there exist two embedded double covers* $\Gamma^{(m)}$ *and* $\Gamma^{[m]}$ *of complete graph* K_n, $n = \binom{m}{2}$, *on* 2^{m-1} *and* 2^{m-2} *vertices, respectively, and with covering radius* $m/2$ *and* $\lfloor m/4 \rfloor$, *respectively.*

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The intersection arrays of graphs $\Gamma^{(m)}$ and $\Gamma^{[m]}$ are the same as the intersection arrays of codes, given by Theorems 7 and 8.

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Both graphs $\Gamma^{(m)}$ and $\Gamma^{[m]}$ are distance transitive.
The graphs $\Gamma^{(m)}$ are imprimitive and the graphs $\Gamma^{[m]}$ are primitive.
The graph $\Gamma^{[m]}$ has eigenvalues $\{\frac{(m-4i)^2-m}{2}: \; i = 0, 1, \ldots, \rho\}$.

The graph $\Gamma^{(m)}$ is well known. It can can be obtained from the even weight binary vectors of length m, adjacent when their distance is 2. It is the halved m-cube and is a distance-transitive graph, uniquely defined from its intersection array (Brouwer et alt. [1989]).

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Since the graph $\Gamma^{(m)}$ is antipodal, the graph $\Gamma^{[m]}$ (which has twice less vertices) can be seen as its folded graph, obtained by merging antipodal pairs of vertices. It is a halved folded m-cube and it is not determined from its intersection array (Brouwer et alt. [1989], p. 264).

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