A Multiple Access System for a Disjunctive Vector Channel

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Outline

Task statement

Our task is to propose a signal-code construction using A-channel and to study the properties of multiple access system built on the basis of this construction.

A-channel

Let us denote the number of active users by S, $S \geqslant 2$.

Input at time t

Vectors $\mathbf{x}_i^{(t)} \in \{0,1\}^q, |\mathbf{x}_i^{(t)}| = 1,\ i = 1, \ldots, S$.

Output at time t

$$\mathbf{y}^{(t)} = \bigvee_{i=1\ldots S} \mathbf{x}_i^{(t)}$$

The channel is noiseless.

Transmission

Each user encodes the information transmitted by q-ary (n, k, d)-code C (all users use the same code). Consider the process of sending the message by i-th user.

Let us denote the codeword to be transmitted by c_i, each symbol c_i is associated with a binary vector of length q and weight 1, the unit is in a position corresponding to the element of $GF(q)$ to be transmitted. We denote the matrix constructed in this way by $\mathbf{C}_i$.

Transmission is performed symbol by symbol. Before sending a binary vector a random permutation of its elements is performed. The permutations used are selected independently and with equal probability.

Transmission

Example

Let $q = 3,\ C = \{(0,0,0,0),(1,1,1,1),(2,2,2,2)\},\ c_i = (1,1,1,1).$

Let the mapping$(GF(q) \rightarrow \{0,1\}^q)$ be defined in such a way:
$0 \rightarrow (100)^T$, $1 \rightarrow (010)^T$, $2 \rightarrow (001)^T$, then

$$\mathbf{C}_i = \begin{matrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Reception

The base station sequentially receives messages from all users. Let us consider the process of receiving a message from the i-th user. At receiving of each column the reverse permutation is performed. Thus, we obtain the matrix

$$\mathbf{Y}_i = \mathbf{C}_i \vee \left(\bigvee_{m=1\ldots S, m\neq i} \mathbf{X}_m \right),$$

where $\mathbf{C}_i$ is a matrix corresponding to c_i and matrixes $\mathbf{X}_m, m = 1\ldots S, m \neq i$ are the results of another users activity.

Reception

For all $c_t \in C$

1. construct a matrix $\mathbf{C}_t$ corresponding to c_t.
2. if the condition $\mathbf{C}_t \wedge \mathbf{Y}_i = \mathbf{C}_t$ follows add c_t to a list of possible codewords.
3. go to next c_t.

In case of only one word in the list output the word, else output a denial of decoding.

Probability of denial

Theorem

The estimate follows

$$p_* \leqslant \sum_{W=d}^{n} \left[A(W) \left(1 - \left(1 - \frac{1}{q} \right)^{S-1} \right)^{W} \right] <$$

$$< q^k \left(1 - \left(1 - \frac{1}{q} \right)^{S-1} \right)^{d},$$

where $A(W)$ is a number of codewords of weight W in a code C.

Probability of denial

Corollary

Let q, k, S and p_r be fixed, than if the condition

$$d \geqslant \frac{k - \log_q p_r}{\beta},$$

follows, where $\beta = -\log_q \left(1 - \left(1 - \frac{1}{q}\right)^{S-1}\right)$, *than* $p_* < p_r$

Definitions

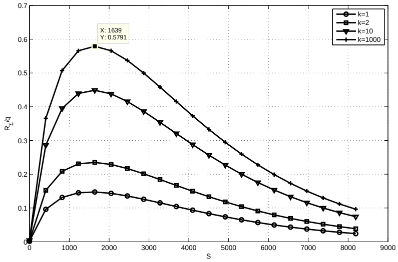
The rate for one user

$$R_i(q, S, k, c) = \frac{k}{n(q, S, k, c)} \log_2 q.$$

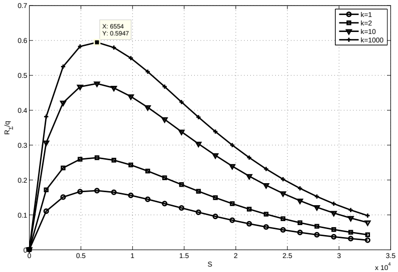
Group rate

$$R_\Sigma(q, S, k, c) = S \frac{k}{n(q, S, k, c)} \log_2 q.$$

Dependencies of group rate on number of active users



$q = 2^{11},\ p_r = 10^{-10}$



$q = 2^{13},\ p_r = 10^{-10}$

Definitions

Relative number of users

$$\gamma = S/q.$$

Relative asymptotic group rate ($p_r = 2^{-cn}, c > 0$)

$$\rho(\gamma, k, c) = \lim_{q \to \infty} \frac{R_\Sigma(q, \gamma q, k, c)}{q}.$$

An asymptotic estimate of group rate

Theorem

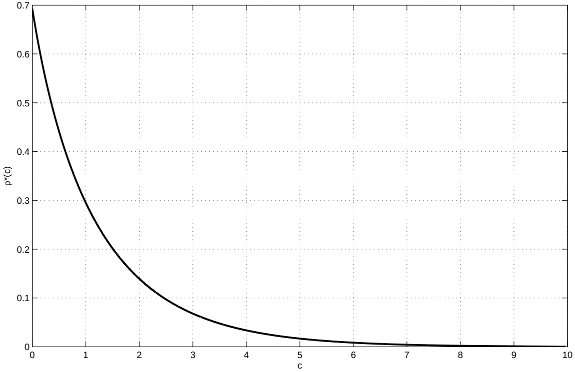
If $\gamma < -\ln\left(1 - 2^{-c}\right)$ *than the following inequality follows*

$$\rho\left(\gamma, k, c\right) \geqslant \underline{\rho}(\gamma, c) = \gamma\left(\log_2\left(\frac{1}{1 - e^{-\gamma}}\right) - c\right).$$

Let us introduce a notion

$$\rho^*(c) = \max_{\gamma}\left[\underline{\rho}(\gamma, c)\right].$$

The dependency of $\rho^*(c)$ on c



Note that $\rho^*(\varepsilon) \geqslant (1-\varepsilon)\ln 2 = (1-\varepsilon)0,693\ldots$.

Conclusion

Main results:

1. A novel signal-code construction has been proposed. The construction does not need block synchronization.
2. A lower bound on a group rate in the multiple access system built on the basis of this construction is derived. The bound coincides with an upper bound in case of $c = \varepsilon$.

Thank you for the attention!