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On codes for flash memory

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Abstract. The problems of writing data and error correction for flash memories are discussed.

1 Introduction

Nonvolatile memory is computer memory that maintains stored information without a power supply. With the rise of portable electronic devices like cell phones, mp3 players, digital cameras, and PDAs, nonvolatile memory is increasingly important. Flash memory is currently the dominant nonvolatile memory. It is cheap because it does not contain any moving parts, consumes less power, and can be electrically programmed and erased with relative ease. Reading and writing are very fast (~ 100 times faster than disk).

Flash memory consists of cells that store one or more bits by electrical charge of two or more voltage levels. The cells are organized in blocks. Each block stores ~ 512 KB. Flash memory has two very specific features:

- The voltage of the charge can easily be increased, but can only be decreased by an erasure operation. Only whole blocks can be erased.
- Erasures are very slow. Each block has a limited number of erase cycles it can handle. After 10,000 - 100,000 erasures, the block cannot be reliably used.

Most recently used flash memory stores 1 or 2 bits per cell (see Figure 1) but there are proposals for up 8 bits (i.e. 256 voltage levels).

Physical characteristics of flash memory, namely impossibility to decrease the voltage only in a cell results in asymmetry both in reading and writing processes. This leads to two main problems concerning flash memories

(R) Development of suitable error correcting codes. Errors occur during the process of reading are with limited magnitude and in one dominant direction (the "read voltage level" is less than the actual one). This is due to the natural electrons' leaking out and the 2D intersymbol interference.

(W) Development of methods for writing in (re-programming) cells with as minimum as possible erasures (WOM codes, floating codes, flash code, rewriting codes, etc.).

For solving problem (R), that is, for correcting asymmetric errors, conventional (symmetric) error correcting codes as BCH [7, 9], Reed-Solomon [3], LDPC codes [8] was first used. Later codes over ring $\mathbb{Z}_q$ of integers modulo q specially constructed for correcting asymmetric errors have been proposed. Cassuto et al. in [2], Klove et al. in [4], [5], etc., have done thorough study of q-ary codes for asymmetric error.

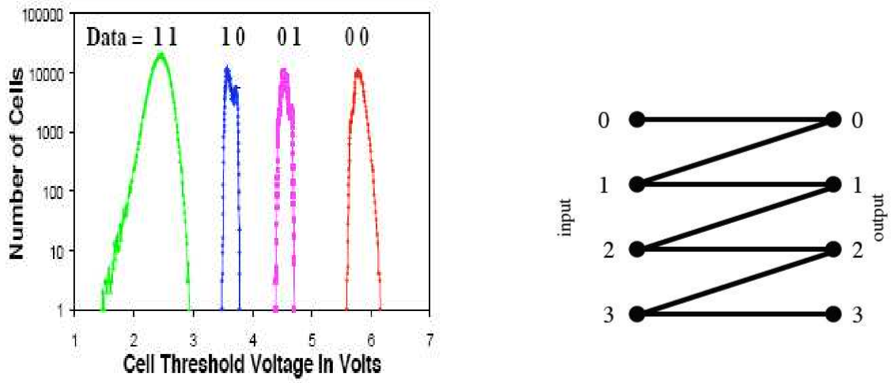

Figure 1: Left:The voltage distribution in four level cell. Right: A q-ary asymmetric 1-limited-magnitude error channel. [6]

In Section 2 we give necessary definitions. A new general construction of single asymmetric limited magnitude error correcting integer codes will be presented in Section 3. Conclusion remarks are given in Section 4.

2 Limited magnitude error-correcting codes

2.1 Notations and definitions

Asymmetric error correcting codes have a long history. First they were considered in the middle of sixties by Varshamov and Tenengolz [11]. In 1973 Varshamov [12] introduced a q-ary asymmetric channel. Multilevel flash memories renew interest in codes correcting asymmetric errors. The q level voltage charge in a cell are well describe by elements of the ring $\mathbb{Z}_q$ of integers modulo q. At ACCT'02 in 2002 R. Ahlswede, H. Aydinian and L. Khachatrian [1] introduced a q-ary asymmetric channel that slightly differ from the Varshamov's channel. It is described as follows.

Let $\mathcal{A}$ be an alphabet of size q, for example $\mathcal{A} = GF(q)$ or $\mathbb{Z}_q$. Consider codes of block length n, that is, subsets $C \subset \mathcal{A}^n$. Let a codeword $\mathbf{x} = (x_1, x_2, \dots, x_n)$ be sent through a channel or stored in a memory. During transmission or in the process of reading from the memory $\mathbf{x}$ is transformed into $\mathbf{y} = \mathbf{x} + \mathbf{e}$, where $\mathbf{e} = (e_1, e_2, \dots, e_n) \in \mathcal{A}^n$ is called an error vector.

Definition 1. *An error vector $\mathbf{e} = (e_1, e_2, \dots, e_n)$ is called a **t-asymmetric λ-limited-magnitude error** if $\mathrm{wt}(\mathbf{e}) = |\{i : e_i \neq 0\}| \leq t$ and $0 \leq e_i \leq \lambda$, for all $i = 1, 2, \dots, n$. A code C is called a **t-asymmetric λ-limited-magnitude error correcting code** if it can correct all t-asymmetric λ-limited-magnitude errors.*

Let $E(\lambda, n, t)$ denote the set of all possible t asymmetric λ-limited-magnitude error vectors of length n over $\mathcal{A}$. Its number of elements is

$$|E(\lambda, n, t)| = \sum_{i=1}^{t} \binom{n}{i} \lambda^i.$$

Let $\mathcal{A} = \mathbb{Z}_q$. Herein, a q-ary ***integer code*** of length n with parity check matrix $\mathbf{H} \in \mathbb{Z}_q^{r \times n}$, is referred to be the subset of $\mathbb{Z}_q^n$, defined by

$$\mathcal{C}(\mathbf{H}, \mathbf{d}) = \{\mathbf{c} \in \mathbb{Z}_q^n \mid \mathbf{c}\mathbf{H}^T = \mathbf{d} \pmod{q}\}$$

where $\mathbf{d} \in \mathbb{Z}_q^r$. If $\mathbf{d} = \mathbf{0}$ the code is a linear code over $\mathbb{Z}_q$. We will write $\mathcal{C}(\mathbf{H})$, or only $\mathcal{C}$ if there is no possibility for ambiguity.

Proposition 1. *The code $\mathcal{C}(\mathbf{H}, \mathbf{d})$ is a t-asymmetric λ-limited-magnitude error correcting code if syndromes of all elements of $E(\lambda, n, t)$ are distinct, that is, all the vectors of the set*

$$\left\{\mathbf{e}\mathbf{H}^T \mid \mathbf{e} \in E(\lambda, n, t)\right\}$$

are distinct.

Proposition 2 (Hamming bound)**.** *If $\mathcal{C}(\mathbf{H})$ is a q-ary t-asymmetric λ-limited-magnitude error correcting code then*

$$q^r \geq \sum_{i=0}^{t} \binom{n}{i} \lambda^i. \tag{1}$$

Without loss of generality we shall assume that $\mathbf{d} = \mathbf{0} \in \mathbb{Z}_q^r$.

Definition 2. *A q-ary t-asymmetric λ-limited-magnitude error correcting code $\mathcal{C}(\mathbf{H}, \mathbf{d})$ of block length n is called **perfect**, when we have equality in (1).*

The case $r = 1$, i.e., when $\mathbf{H}$ is a row of length n, is mainly discussed in the literature.

2.2 Several proposed q-ary codes

In [2] Cassuto et al. describe a general method of constructing t-asymmetric λ-limited-magnitude error correcting codes from codes correcting symmetric errors.

Recently T. Klove et al. in [4] and [5] have done thorough study of t-asymmetric λ-limited-magnitude error correcting codes over $\mathbb{Z}_q$. Their study is based on the fact that the discussed coding problems can be reformulated and solve as problems in number theory. In the cited papers the notation $B_t[\lambda](q)$ is used, or just $B_t[\lambda]$ when q is known from the context. Namely, $B_t[\lambda](q)$ is defined as a set $B_t[\lambda](q) = \{b_1, b_2, \dots, b_n\}$ such that the set

$$\mathbf{e}B_t[\lambda](q) = \{\ e_1b_1 + e_2b_2 + \cdots + e_nb_n \mid \mathbf{e} \in E(\lambda, n, t)\}$$

consists of distinct elements of $\mathbb{Z}_q$, i.e., modulo q. In these papers classes of codes correcting $t = n$ and $t = n - 1$ asymmetric λ-limited-magnitude errors are proposed. But the most attention was paid to the case $t = 1$, i.e., the set $B_1[\lambda](q)$. The Hamming bound for such codes gives $q \geq 1 + \lambda n$.

Define $M_\lambda(q)$ to be the maximal size of a $B_1[l](q)$ set. In [4] it has been shown that for odd values of q we have

$$M_\lambda(q) = \frac{q-1}{2} - \frac{\omega_q}{2}$$

where ω_q is the number of the cyclotomic cosets of odd size. In [5] $M_2(q)$ and bounds for $M_3(q)$ and $M_4(q)$ are determined.

In [5] a perfect $B_1[\lambda](p)$ sets for a class of primes p is described. Also some results about $B_1[\lambda](q)$, $\lambda = 3, 4$, are obtained. Unfortunately theoretical results gives good codes for very large values of q. Optimal for codes over reasonable large alphabets are found by computer search in the case $t = 2$ and $t = n - 2$ for small n.

3 A new construction of single asymmetric limited magnitude codes

In the following Theorem we propose a construction of a single asymmetric 2-limited-magnitude error correctable code.

Proposition 3. *A 1 asymmetric 2-limited-magnitude error correctable code C of length n over Z_q has the following parity-check matrix H*

- $H = (1, 3, 5, \dots, n-1, n+3, n+5, \dots, 2n+1)$, *where* $q = 2n+2$ *and* n *is even*
- $H = (1, 3, 5, \dots, n-3, n+3, n+5, \dots, 2n+1)$, *where* $q = 2n+4$ *and* n *is odd*

Remark. In the case when n is even the code is quasi perfect - the exceeding is 1.

Proof. Here we are going to prove the case when n is even and $q = 2n+2$. The proof when n is odd is analogous.

To show that a code C with parity-check matrix $H = (1, 3, 5, \ldots, n-1, n+3, n+5, \ldots, 2n+1)$ is 1 asymmetric 2-limited-magnitude error correctable it is enough to prove that all the elements in the set $H_1 = 2H \mod (2n+2)$ are distinct and $H \cap H_1 \neq \emptyset$. We have

$$2H = (2, 6, 10, \ldots, 2n-6, 2n-2, 2n+6, 2n+10, \ldots, 4n-2, 4n+2)$$

and

$$H_1 = (2, 6, 10, \ldots, 2n-6, 2n-2, 4, 8, \ldots, 2n-4, 2n).$$

It is not so difficult one to see that all the elements in H_1 are distinct. Moreover, the elements of H_1 are even, while the elements of H are odd. So we have $H \cap H_1 \neq \emptyset$. With that the proof is completed.

$\square$

We would like to note that the construction in Proposition 3 gives codes over $\mathbb{Z}_q$, $q = 2n+2$, that cannot be obtained by the results in [5]

4 Conclusions

In this paper, we showed a construction of a single asymmetric 2-limited magnitude error correctable code. For some parameters, the codes we obtained by this construction are optimal. One can see that we only consider the case of single error and small magnitude. Actually, it is very difficult to obtain theoretical results for multiple errors and higher magnitude. On that we will focus for our future research.

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