New iterative decoder for product codes

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ALGEBRAIC AND COMBINATORIAL CODING THEORY (ACCT-XIV)

Problem solved

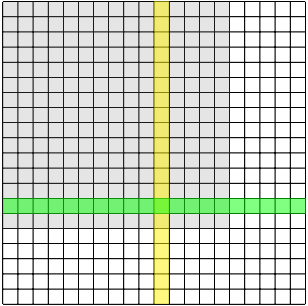
Iterative decoder of product codes has some simple uncorrectable error sets.

In this work we will estimate the probability of this sets appearing (and thus setting a lower bound on error rate) and design a new decoder that can correct some of these sets.

Design of Product Codes

A codeword of a product code is a matrix whose rows are codewords of the first component code and whose columns are codewords of the second component code.

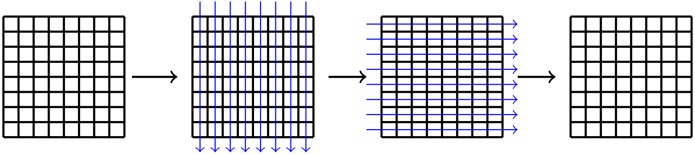
$[n_r, k_r, d_r]_q$



$[n_c, k_c, d_c]_q$

$[n_c n_r, k_c k_r, d_c d_r]_q$

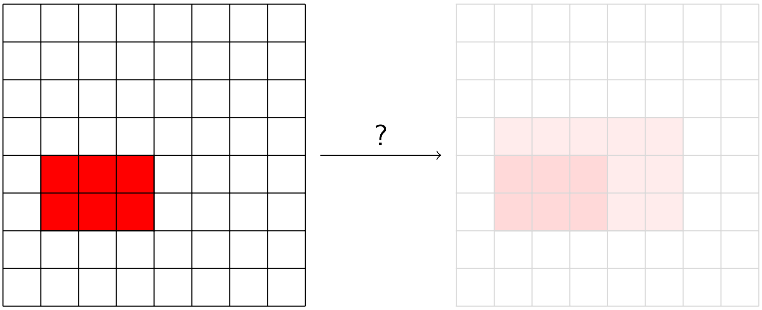
Iterative Decoder



1. Decode all column codes in-place.
2. Decode all row codes in-place.
3. If the resulting word differs from the initial for the current iteration, repeat.

Uncorrectible error set

Let us consider $[8, 4, 5] \times [8, 6, 3]$ product code.



Initial errors

Inserted errors

Uncorrectible error set

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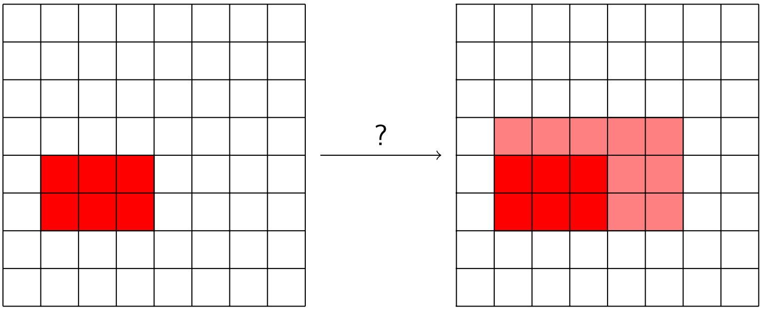

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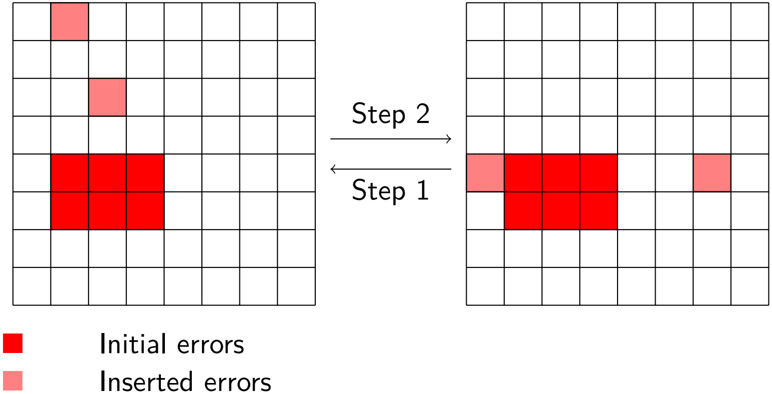
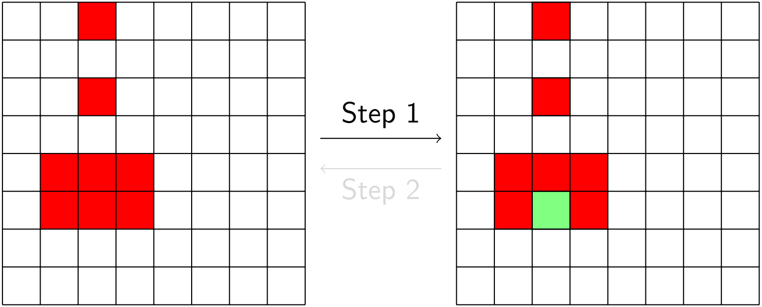


Correctible error set

Let us consider $[8, 4, 5] \times [8, 6, 3]$ product code.



Initial errors

Corrected errors

Correctible error set

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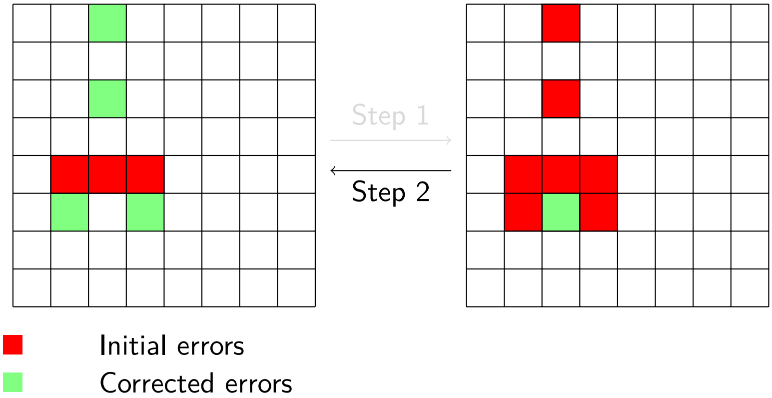

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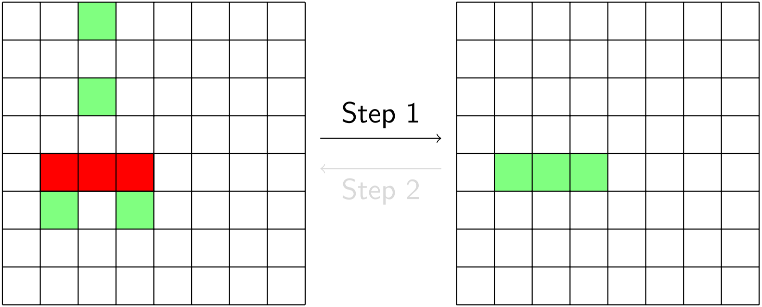

Initial errors

Corrected errors

Criterion

- All the words that have a "rectangle" of errors, some errors outside of its columns and at most one additional error in each of its columns would be decoded incorrectly.
- One additional error in a column of the "rectangle" is not enough to cause correct decoding.
- To count each error set only once we will limit the amount of the additional errors by $\lceil \frac{d_c}{2} \rceil$ if $d_c < d_r$.

$$
\begin{aligned}
P_f \geq & \binom{n_r}{e_r}\binom{n_c}{e_c} p^{e_r e_c}(1-p)^{n_r n_c - e_r e_c} \times \\
& \times \left(\sum_{t=0}^{e_c} \sum_{w=0}^{t} \binom{n_r n_c - e_r n_c}{t-w} \binom{e_r}{w} (n_c - e_c)^w \left(\frac{p}{1-p}\right)^t - \right. \\
& \left. - (n_r - e_r) \left(\frac{p}{1-p}\right)^{e_c} \right)
\end{aligned}
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\end{aligned}
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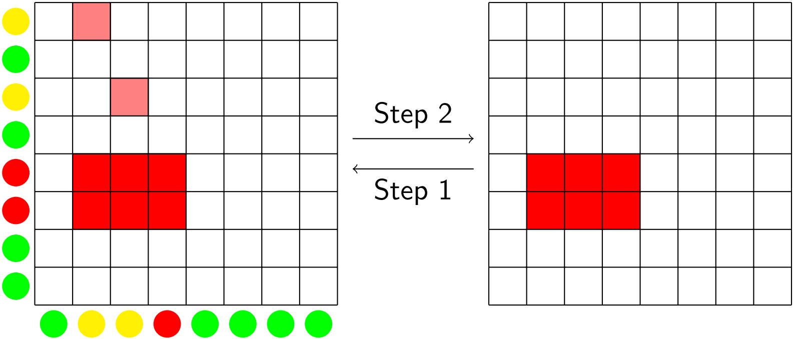
The Proposed Decoder

- After the iterative decoder stops we can try to guess where the dense error submatrix is.
- Let us insert erasures to the guessed submatrix and continue iterative decoding.
- If we guessed right, the decoder would be able to correct all the errors.

How to Guess the Error Submatrix Position?



- No errors detected
- Rejection
- Corrected some errors
- Inserted erasures

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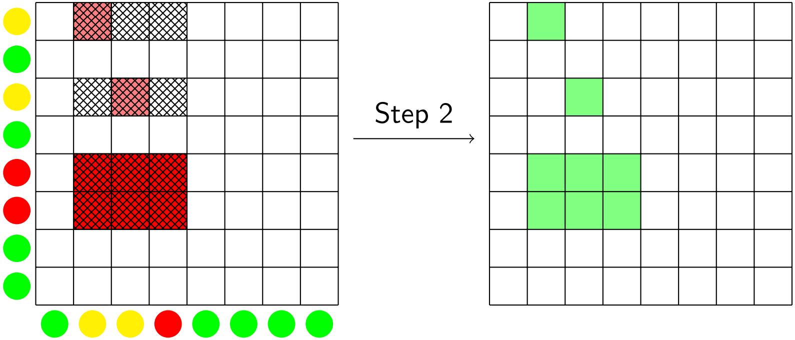
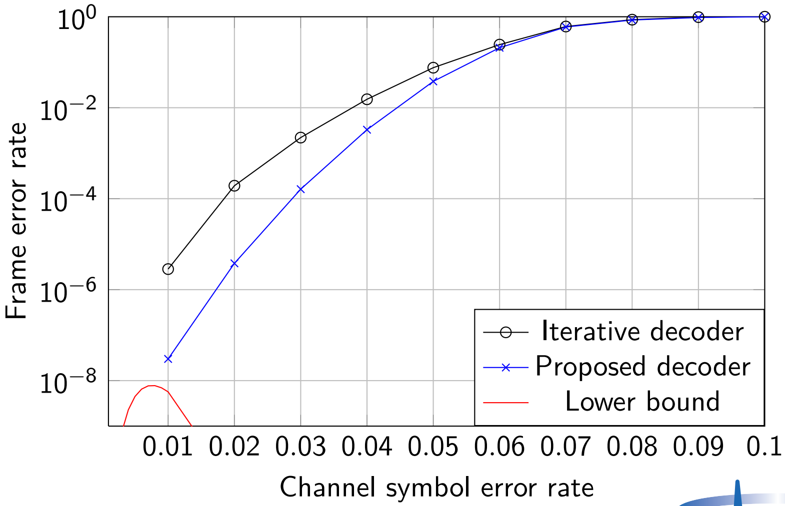


- No errors detected (green)
- Rejection (red)
- Corrected some errors (yellow)
- Inserted erasures (crosshatched)

Simulation parameters

- A product of $[32, 30, 3]_{256}$ and $[32, 28, 5]_{256}$ Reed-Solomon codes.
- This code has length 1024, distance 15 and 840 information symbols.
- The rate is 0.82.
- The channel was a q-ary symmetric channel.
- The uncorrectible error submatrix has size 3×5.

Simulation



Frame error rate
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1
Channel symbol error rate
Iterative decoder
Proposed decoder
Lower bound

Thank you for your attention!