Fourteenth International Workshop on Algebraic and Combinatorial Coding Theory
September 7–13, 2014, Svetlogorsk (Kaliningrad region), Russia *pp. 283–288*

Order statistics-based decoder with reliability information in a multiple access system under interference [1]

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Abstract. Future generation wireless communication systems will have to operate within crowded bands. Thus transmission and reception techniques that can withstand severe interference are of great interest. Recently several jamming-proof signal-code constructions based on coded Dynamic Hopset Allocation OFDMA (DHA OFDMA) were proposed. Hereinafter a modified version of an order statistics-based convolutional decoder (detector) is proposed. The proposed decoder makes use of the path metrics to obtain reliability information. If the computed reliability is too low a denial decision is taken. The effectiveness of the proposed approach is verified by means of simulation of the communication system employing concatenated construction with the proposed detector serving as an inner decoder.

1 Introduction

As the number of communication systems using wireless channels grows drastically future generation communication systems are bound to operate within crowded frequency bands. Thus such systems will have to withstand severe interference from other communication systems (both narrowband and broadband). This problem can be interpreted in terms of combating jamming (partial band jamming and multitone jamming respectively). In [2] a robust detector employing order statistics to decode an inner block code has been proposed. It has been shown that the detector in question can withstand severe partial-band interference. In [4] the same approach (i.e. using order statistics to decode inner codes in a jamming environment) has been used to decode inner convolutional code. It has been demonstrated that the approach in question can provide reliable communication in a multiple access system even under severe interference cased by both narrowband and broadband signals.Hereinafter a modified version of the order statistics-based detector from [4] serving as an inner code decoder in a coded Dynamic Hopset Allocation OFDMA (DHA OFDMA) [3] is proposed. The proposed detector makes use of the path metrics computed by the convolutional decoder to obtain reliability information. If the computed

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[1] This work has been supported by the RFBR (research project No. 14-07-31197 mol_a)

reliability is too low a denial decision is taken. In what follows it will be demonstrated that the use of the proposed modified detector as an inner decoder in a concatenated code construction results in substantial gain.

2 Transmission and Reception in a coded DHA FH OFDMA

Let us consider a multiple access system in which U_A active users transmit information via a channel split into Q identical nonoverlapping subchannels by means of OFDM. Information that is to be transmitted is encoded into a codeword of a $C\left(n,k,d\right)$ q-ary code ($q \ll Q$).Whenever a user is to transmit a $q-$ary symbol a nonzero entry is to be placed in the position of the vector $\bar{a}_g$ corresponding to the symbol in question within the scope of the mapping in use (in what follows it will be assumed that all the positions of the vector are enumerated from 1 to Q, moreover without loss of generality we shall assume that the 1st subchannel corresponds to 0, the 2 nd subchannel corresponds to 1 and so on). Then a random permutation of the aforesaid vector is performed and the resulting vector $\pi_g\left(\bar{a}_g\right)$ is used to form an OFDM symbol (permutations are selected equiprobably from the set of all possible permutations and the choice is performed whenever a symbol is to be transmitted). Therefore in order to transmit a codeword a user is to transmit n OFDM symbols.

Within the scope of reception of a certain codeword the receiver is to receive n OFDM symbols corresponding to the codeword in question. Note that the receiver is assumed to be synchronized with the transmitters of all users. Therefore all the permutations done within the scope of transmission of the codeword in question are known to the user. The receiver measures energies at the outputs of all subchannels (let us designate the vector of the measurements as b_g where g is the number of the OFDM symbol) and applies inverse permutation to each vector b_g corresponding to the respective OFDM symbol thus reconstructing the initial order of elements and obtaining vector $\tilde{b}_g = \pi_g^{-1}\left(b_g\right)$. Let us consider a matrix X that consists of vectors $\tilde{b}_g$ that correspond to the codeword transmitted by the user under consideration. Let us note that submatrix $\tilde{X}_q$ (that consists of the q first rows of matrix X) corresponds to the received codeword. The detector is to decide on the transmitted codeword by analyzing the matrix $\tilde{X}_q$.

3 Order statistics-based receiver

To decode the codeword corresponding to the matrix $\tilde{X}_q$ the receiver is to compute reliability values for all codewords of the code C (thus the decoding algorithm in question boils down to exhaustive search). The most popular solution is to use information on the distribution of the decision statistics to compute reliability values of the symbols (e.g. probabilities or LLR's) i.e. to use maximum likelihood (ML) decoding. Unfortunately under severe jamming this approach is hardly feasible since interfering signals with energy much higher than that of the signal transmitted by the user under consideration can drastically change reliability values of the symbols thus leading to erroneous decoding. Consequently a more robust metric is needed.

In [2] an order statistics-based metric has been proposed to enable reliable communications even in "bad" channels.It associates each element of the matrix $\tilde{X}_q$ with its rank i.e. number of elements that are less than the corresponding element. Rank can be formally defined in terms of indicator function

$$I(x^*, x) = \begin{cases} 1 & x \leq x^* \\ 0 & x > x^* \end{cases} \tag{1}$$

rank of the element x_{ij} is given by

$$\rho(x_{ij}) = \sum_{k \neq i} \sum_{m \neq j} I\left(x_{ij}, x_{km}\right) \tag{2}$$

Within the scope of the decoding algorithm proposed in [2] each codeword is associated with the sum of ranks assigned to the symbols of the respective codeword. Let us designate the mapping in use with M. Thus each element $c_i(m)$ of the $i-th$ codeword is mapped into an element of matrix $\tilde{X}_q(m, M(c_i(m)))$. The decoding rule boils down to choosing

$$\bar{c}^* = \arg\max_{\bar{c}_i \in C} \Re\left(\bar{c}\right) \ = \arg\max_{\bar{c}_i \in C} \sum_{m=1}^{n} \rho\left(\tilde{X}_q(m, M(c_i(m)))\right) \tag{3}$$

Please note that both the length and the minimum distance of the block code in use are to be great in order to provide acceptable probabilistic characteristics. However since the decoding algorithm described above boils down to exhaustive search the complexity of the decoder depends on the codeword length. Thus to make the decoding computationally feasible one has to use low rate inner codes.

In order to raise the transmission rate inner nonbinary convolutional codes [4] were proposed. Such codes can be decoded with the Viterbi algorithm [1] employing rank metric. It is well known that the complexity of the Viterbi decoder (whatever metric is used) is exponential with the overall constraint length but linear with the code length. Thus nonbinary convolutional codes with small overall constraint lengths can provide relatively high transmission rates with reasonable computational complexity.Let us consider a q-ary convolutional code with rate b/c, memory m and s_i information symbols (the length of the resulting codeword is therefore $l = s_i \cdot c$) constructed in the following way: the information symbols and the content of the shift register are encoded with systematic $(q-1, m+b)$ Reed-Solomon code (further on this code will be referred to as a component code). Each tuple consists of the c parity check symbols of the component code.To preserve the inner code rate we shall use Direct Truncation technique.

4 Modified receiver

The detector (decoder) described above is a hard decision detector, i.e. it chooses the codeword that corresponds to the maximum rank sum. However rank sums can be considered as estimates of each codeword reliability. Therefore if a concatenated code construction is used the decoder of the outer code can benefit from using reliability information obtained from the inner code rank decoder.

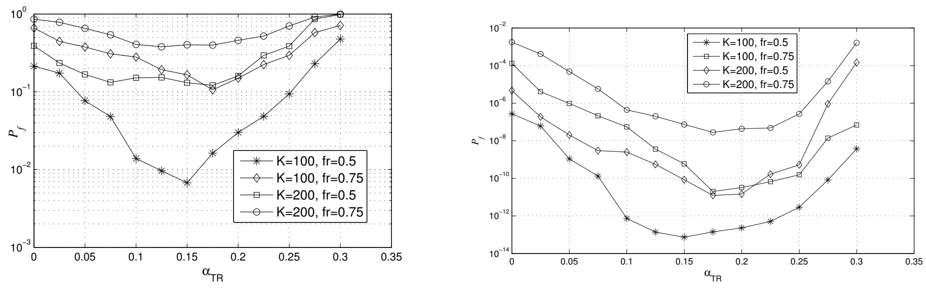
Let us consider the values of the rank sums computed for different paths of the trellis.Let us assume a length B vector of rank sums $\Re_i, i = 1 \dots B$. Let us consider a vector $\tilde{\Re}$ which is the same vector of rank sums but sorted in the descending order. Even though the rank sum is not (in a rigorous mathematical sense) a distance metric it is the difference between the "winning" and the "second best" path that predetermines the reliability of the decision made by the decoder. I.e. if the difference is "small" the decision can hardly be assumed reliable. However since different scenarios correspond to different matrixes $\tilde{X}_q$ this value is to be normalized in order to provide better estimate of the codeword reliability. Therefore hereinafter we shall consider the value

$$\alpha = \frac{\tilde{\Re}_1 - \tilde{\Re}_2}{\tilde{\Re}_2 - \tilde{\Re}_B} \tag{4}$$

If the value of this parameter is "small" the best path is too close to other paths of the trellis to consider it reliable. Hereinafter we are aiming at finding such a critical value α_{TR} that if $\alpha \leq \alpha_{TR}$ the decision of the Viterbi decoder is assumed unreliable and therefore a denial decision is to be taken instead.

5 Simulation and parameters choice

Let us now consider a real-life scenario which will be used in order to verify the applicability of the proposed approach. An OFDM system with $N = 4096$ subcarriers has been considered (3276 being available to the users). Lognormal path loss model with the typical distance $d_0 = 100\ m$ and the radius of the cell $d_{\max} = 2\ km$ was considered. A "pessimistic" simulation scenario has been used: within the scope of this scenario the user under consideration was assumed to be at the edge of the cell, whereas each of the interfering users was assumed to be at a distance of d (where d is equiprobably equal to $[d_0, 2d_0, 3d_0]$, distances being chosen at each instance. Please note that no power control has



(a) RS(240,200) (b) RS(240,160)

Figure 1: Dependencies of the probability of false decoding on the value of the parameter α_{TR} for various outer code parameters

been considered within the system under consideration, i.e. the signals from the interfering users (at the receiver end) have powers much greater than that of the signal from the user under consideration. As for the channel model under consideration a time-varying Raleigh fading channel 802.11 b model has been used.

The number of interfering users is equal to K. It has been assumed that the received signal is affected by both background noise (denoted by the SNR value per bit) and partial-band noise. The fraction of the effective bandwidth that is affected by the partial-band noise is given by $fr(0 < fr \leq 1)$ while the energy at the receiver end is given by the signal to interference ratio SIR per transmitted bit. Hereinafter we shall assume that a convolutional code with memory $m = 1$ overall constraint length $\upsilon = 2$ and rate $R = 2/10$ over $GF(16)$ obtained from a (15,4,12) Reed-Solomon code is used as an inner code.

To determine the optimal value of the parameter α_{TR} simulation has been used. It has been assumed that the convolutional code under consideration is used as an inner code in a product-code like construction. Outer Reed-solomon codes of different rates over $GF(256)$ were considered. In Fig. 1 examples of the obtained curves for the probability of false decision (i.e. the probability of the fact that decoding will result in either denial or error) for various values of fr and $SIR = -25dB$,$SNR = 5dB$ and $K = 200$ are shown

As can be seen "optimal" value of the parameter α is within the range $0.15 \ldots 0.175$. Let us now consider an example of the modified detector application. Assume for instance the following product code construction: outer (horizontal) code is a Reed-Solomon code $(240, 160)$ over $GF(256)$. Inner code is a length $L = 12$ rate $R = 2/10$ convolutional code obtained from Reed-Solomon code (15,4,12). Denial probabilities for various values of fr, and $SIR = -25dB$,$SNR = 5dB$ and $K = 200$ are given in Table 1 As can be seen for the parameters under consideration the usage of the conventional decoder results in unacceptably high denial probabilities for the outer code. Thus to

Table 1: Performance comparison

fr	0,75	1
Decoder type	Denial probability	
No reliability information	$8.5 \cdot 10^{-2}$	$1.6 \cdot 10^{-1}$
$\alpha_{TR} = 0.175$	$6.3 \cdot 10^{-7}$	$4.8 \cdot 10^{-6}$

make use of the conventional detector a lower rate outer code is to be employed. However employment of the modified detector leads to low denial probabilities thus making the proposed construction applicable.

6 Conclusion

In what follows a modified order statistics-based detector has been proposed. Similar to the one proposed in [4] it employs Viterbi algorithm with rank metric to find the path corresponding to the transmitted codeword. However unlike both the convolutional decoder from [4] and block decoder from [2] it employs the obtained metrics for paths of the trellis to determine the reliability of the obtained decision. If the computed reliability value is below the predetermined threshold the decoding results in denial. Due to improved characteristics of the inner decoder higher rate outer codes can be used to ensure the desired performance.

References

[1] A.J. Viterbi: Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm // IEEE Trans. Information Theory, IT-13:260-69, April 1967

[2] K. Kondrashov, V. Afanassiev Ordered statistics decoding for semi-orthogonal linear block codes over random non-Gaussian channels In proc. of the Thirteenth International Workshop on Algebraic and Combinatorial Coding Theory June 15-21, 2012, Pomorie, Bulgaria pp. 192-196,

[3] D. Osipov, A. Frolov, V. Zyablov. Multiple Access System for a Vector Disjunctive Channel. Problems of Information Transmission, vol. 48. no. 3 pp. 243–249, 2012.

[4] D. Osipov Inner Convolutional Codes and Ordered Statistics Decoding in a Multiple Access System Enabling Wireless Coexistence Proceedings of the 6th International Workshop, MACOM 2013, Vilnius, Lithuania, December 16-17, 2013. pp 33-38