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Nonexistence of some Griesmer codes of dimension 4 over $\mathbb{F}_q$ [1]

Kazuki Kumegawa su301015@edu.osakafu-u.ac.jp
Tatsuya Maruta maruta@mi.s.osakafu-u.ac.jp
Department of Mathematics and Information Sciences,
Osaka Prefecture University, Sakai, Osaka 599-8531, Japan

Abstract. We prove the nonexistence of $[g_q(4,d),4,d]_q$ codes for (a) $d = q^3/2-q^2-2q+1$ for $q = 2^h$, $h \geq 3$, (b) $d = 2q^3-3q^2-2q+1$ for $q \geq 7$, and (c) $d = 2q^3-rq^2-q+1$ for $3 \leq r \leq q-q/p$, $q = p^h$ with p prime, where $g_q(k,d) = \sum_{i=0}^{k-1} \lceil d/q^i \rceil$.

1 Introduction

An $[n,k,d]_q$ code $\mathcal{C}$ is a linear code of length n, dimension k and minimum Hamming weight d over $\mathbb{F}_q$, the field of q elements. A fundamental problem in coding theory is to find $n_q(k,d)$, the minimum length n for which an $[n,k,d]_q$ code exists. The Griesmer bound gives a lower bound on $n_q(k,d)$ as $n_q(k,d) \geq g_q(k,d) = \sum_{i=0}^{k-1} \lceil d/q^i \rceil$, where $\lceil x \rceil$ denotes the smallest integer $\geq x$. An $[n,k,d]_q$ code is called *Griesmer* if $n = g_q(k,d)$. The values of $n_q(k,d)$ are determined for all d only for some small values of q and k. For $k = 4$, $n_q(4,d)$ is known for all d only for $q = 2,3,4$. It is known that $n_q(4,d) \geq g_q(4,d)+1$ for $q^3/2-q^2-q+1 \leq d \leq q^3/2-q^2$ for even $q \geq 4$ [4] and for $2q^3-rq^2-q+1 \leq d \leq 2q^3-rq^2$ for $q > r, r = 3,4$ and for $q > 2(r-1), r \geq 5$ [5]. It is also known that $n_q(4,d) = g_q(4,d)+1$ for $2q^3-3q^2-q+1 \leq d \leq 2q^3-3q^2$ for $q \geq 4$ [5]. Our main result is following.

Theorem 1. *There exist no* $[g_q(4,d),4,d]_q$ *codes for*
(a) $d = q^3/2-q^2-2q+1$ *for* $q = 2^h$, $h \geq 3$,
(b) $d = 2q^3-3q^2-2q+1$ *for* $q \geq 7$, *and*
(c) $d = 2q^3-rq^2-q+1$ *for* $3 \leq r \leq q-q/p$, $q = p^h$ *with* p *prime.*

Note that a $[g_q(4,d)+1,4,d]_q$ code with $d = 2q^3-3q^2-q$ can be constructed, see Lemma 8 in [3]. Since the existence of an $[n,k,d]_q$ code implies the existence of an $[n-1,k,d-1]_q$ code, we get the following results (combining the previously known results).

Corollary 2. $n_q(4,d) = g_q(4,d)+1$ *for* $2q^3-3q^2-2q+1 \leq d \leq 2q^3-3q^2$ *for* $q \geq 5$.

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Corollary 3. *$n_q(4,d) \geq g_q(4,d)+1$ for*
(a) *$q^3/2 - q^2 - 2q + 1 \leq d \leq q^3/2 - q^2$ for $q = 2^h$, $h \geq 3$,*
(b) *$2q^3 - rq^2 - q + 1 \leq d \leq 2q^3 - rq^2$ for $4 \leq r \leq q - q/p$, $q = p^h$ with p prime.*

2 Preliminary results

We denote by $\mathrm{PG}(r,q)$ the projective geometry of dimension r over $\mathbb{F}_q$. The 0-flats, 1-flats, 2-flats, $(r-2)$-flats and $(r-1)$-flats in $\mathrm{PG}(r,q)$ are called *points, lines, planes, secundums* and *hyperplanes*, respectively. We denote by $\mathcal{F}_j$ the set of j-flats of $\mathrm{PG}(r,q)$ and by θ_j the number of points in a j-flat, i.e., $\theta_j = (q^{j+1}-1)/(q-1)$. Let $\mathcal{C}$ be an $[n,k,d]_q$ code having no coordinate which is identically zero. The columns of a generator matrix of $\mathcal{C}$ can be considered as a multiset of n points in $\Sigma = \mathrm{PG}(k-1,q)$ denoted also by $\mathcal{C}$. We see linear codes from this geometrical point of view. An *i-point* is a point of Σ which has multiplicity i in $\mathcal{C}$. Denote by γ_0 the maximum multiplicity of a point from Σ in $\mathcal{C}$ and let C_i be the set of i-points in Σ, $0 \leq i \leq \gamma_0$. For any subset S of Σ, *the multiplicity of S with respect to $\mathcal{C}$*, denoted by $m_{\mathcal{C}}(S)$, is defined as $m_{\mathcal{C}}(S) = \sum_{i=1}^{\gamma_0} i \cdot |S \cap C_i|$, where $|T|$ denotes the number of elements in a set T. A line l with $t = m_{\mathcal{C}}(l)$ is called a *t-line*. A *t-plane* and so on are defined similarly. Then we obtain the partition $\Sigma = \bigcup_{i=0}^{\gamma_0} C_i$ such that $n = m_{\mathcal{C}}(\Sigma)$ and $n - d = \max\{m_{\mathcal{C}}(\pi) \mid \pi \in \mathcal{F}_{k-2}\}$. Conversely such a partition $\Sigma = \bigcup_{i=0}^{\gamma_0} C_i$ as above gives an $[n,k,d]_q$ code in the natural manner. For an m-flat Π in Σ, we define $\gamma_j(\Pi) = \max\{m_{\mathcal{C}}(\Delta) \mid \Delta \subset \Pi,\ \Delta \in \mathcal{F}_j\},\ 0 \leq j \leq m$. We denote simply by γ_j instead of $\gamma_j(\Sigma)$. Then $\gamma_{k-2} = n-d,\ \gamma_{k-1} = n$. For a Griesmer $[n,k,d]_q$ code, it is known (see [6]) that

$$\gamma_j = \sum_{u=0}^{j} \lceil d/q^{k-1-u} \rceil \quad \text{for } 0 \leq j \leq k-1. \tag{1}$$

So, every Griesmer $[n,k,d]_q$ code is projective if $d \leq q^{k-1}$. Denote by a_i the number of hyperplanes Π in Σ with $m_{\mathcal{C}}(\Pi) = i$ and by λ_s the number of s-points in Σ. The list of a_i's is called the *spectrum* of $\mathcal{C}$. We usually use τ_j's for the spectrum of a hyperplane of Σ to distinguish from the spectrum of $\mathcal{C}$. Simple counting arguments yield the following.

Lemma 4. (a) $\sum_{i=0}^{n-d} a_i = \theta_{k-1}$. (b) $\sum_{i=1}^{n-d} i a_i = n\theta_{k-2}$.
(c) $\sum_{i=2}^{n-d} i(i-1)a_i = n(n-1)\theta_{k-3} + q^{k-2}\sum_{s=2}^{\gamma_0} s(s-1)\lambda_s$.

When $\gamma_0 \leq 2$, the above three equalities yield the following:

$$\sum_{i=0}^{n-d-2} \binom{n-d-i}{2} a_i = \binom{n-d}{2}\theta_{k-1} - n(n-d-1)\theta_{k-2} + \binom{n}{2}\theta_{k-3} + q^{k-2}\lambda_2. \tag{2}$$

Lemma 5 ([7]). *Let* Π *be an* i*-hyperplane through a* t*-secundum* δ*. Then*
(a) $t \le \gamma_{k-2} - (n-i)/q = (i + q\gamma_{k-2} - n)/q$.
(b) $a_i = 0$ *if an* $[i, k-1, d_0]_q$ *code with* $d_0 \ge i - \lfloor (i + q\gamma_{k-2} - n)/q \rfloor$ *does not exist, where* $\lfloor x \rfloor$ *denotes the largest integer less than or equal to* x.
(c) $\gamma_{k-3}(\Pi) = \lfloor (i + q\gamma_{k-2} - n)/q \rfloor$ *if an* $[i, k-1, d_1]_q$ *code with* $d_1 \ge i - \lfloor (i + q\gamma_{k-2} - n)/q \rfloor + 1$ *does not exist.*
(d) *Let* c_j *be the number of* j*-hyperplanes through* δ *other than* Π*. Then*

$$\sum_j (\gamma_{k-2} - j)c_j = i + q\gamma_{k-2} - n - qt. \tag{3}$$

(e) *For a* γ_{k-2}*-hyperplane* Π_0 *with spectrum* $(\tau_0, \cdots, \tau_{\gamma_{k-3}})$, $\tau_t > 0$ *holds if* $i + q\gamma_{k-2} - n - qt < q$.

Lemma 6 ([4]). *For even* $q \ge 4$*, the spectrum of a* $[q(q-1)/2 - 1, 3, q(q-2)/2 - 1]_q$ *code is* $(a_0, a_{q/2-1}, a_{q/2}) = (q+2, q+1, q^2 - q - 2)$.

Lemma 7 ([4]). *For* $q \ge 7$*, the spectrum of a* $[2q^2 - q - 2, 3, 2q^2 - 3q - 1]_q$ *code is either* $(a_{q-2}, a_{q-1}, a_{2q-2}, a_{2q-1}) = (1, 2, q, q^2 - 2)$ *or* $(a_{q-1}, a_{2q-2}, a_{2q-1}) = (3, q+1, q^2 - 3)$.

An n-set K in $\mathrm{PG}(2, q)$ is an (n, r)-*arc* if every line meets K in at most r points and if some line meets K in exactly r points. Let $m_r(2, q)$ denote the largest value of n for which an (n, r)-arc exists in $\mathrm{PG}(2, q)$.

Lemma 8 ([2]). (a) $m_r(2, q) \le (r-1)q + r$.
(b) $m_r(2, q) \le (r-1)q + r - 3$ *for* $4 \le q < q$ *with* $r \not| q$.
(c) $m_r(2, q) \le (r-1)q + r - 4$ *for* $9 \le q < q$ *with* $r \not| q$.

An f-multiset F in $\mathrm{PG}(2, q)$ is an (f, m)-*minihyper* if every line meets F in at least m points and if some line meets F in exactly m points with multiplicity.

Lemma 9 ([1]). *Every* $(x(q+1), x)$*-minihyper in* $PG(2, q)$*,* $q = p^h$*,* p *prime, with* $x \le q - q/p$ *is a sum of* x *lines.*

3 Proof of Theorem 1

To prove (a) of Theorem 1, let $\mathcal{C}_1$ be a putative $[g_q(4, d), 4, d]_q$ codes for $d = q^3/2 - q^2 - 2q + 1$ for $q = 2^h$, $h \ge 4$ (see [4] for $q = 8$). By Lemma 6, the spectrum of a γ_2-plane Δ_1 is $(\tau_0, \tau_{\frac{q}{2}-1}, \tau_{\frac{q}{2}}) = (q+2, q+1, q^2 - q - 2)$. For any integer i with $1 \le i \le (q^2 - 5q - 2)/2$, we have

$$t \le \frac{i}{q} + \frac{3}{2} \tag{4}$$

by Lemma 5. Let $\mathcal{C}_1^{(i)}$ be the $[i,3]_q$ code corresponding to an i-plane δ_1. For $1 \le i \le \frac{q}{2}-1$, we get $t \le 1$ from (4). Since δ_1 has no 1-line, we have $t=0$ by Lemma 5(e), which is impossible. For $(s-\frac{1}{2})q \le i \le (s+\frac{1}{2})q-1$ $(1 \le s \le \frac{q}{2}-3)$, we get $t \le s+1$ from (4). Since δ_1 has no $(s+1)$-line, we have $t \le s$ and $\mathcal{C}_1^{(i)}$ is an $[i,3,d_0]_q$ code with $d_0 \ge i-s$ by Lemma 5(e), which does not exist by the Griesmer bound. Hence $a_i = 0$ for all $i \notin \{0,(q^2-5q)/2,\cdots,(q^2-q-2)/2\}$. It follows from (2) with $\lambda_2 = 0$ that

$$\binom{\gamma_2}{2}a_0 + \sum_{j=0}^{2q-1}\binom{2q-1-j}{2}a_{\alpha+j} = \frac{q^5+q^4}{8} - \frac{3}{2}q^2 - 2q + 1, \tag{5}$$

where $\alpha = \frac{q^2-5q}{2}$. Lemma 5(d) gives $\sum_j c_j = q$ and

$$\sum_j (\frac{q^2-q-2}{2} - j)c_j = i + \frac{3}{2}q - qt. \tag{6}$$

Suppose $a_0 > 0$. It follows from (6) that $a_0 = 1$ and $a_j = 0$ for $1 \le j \le \frac{q^2-4q-4}{2}$. Setting $i = n-d = \frac{q^2-q-2}{2}$, RHS of (6) is $\frac{q^2+2q-2}{2}$ for $t=0$; $2q-1$ for $t=\frac{q}{2}-1$; $q-1$ for $t=\frac{q}{2}$. If $q \equiv 2 \bmod 3$, the maximum possible contributions of c_j's in (6) to the LHS of (5) are $(c_0, c_{\frac{q^2-4q-2}{2}}, c_{n-d}) = (1,1,q-2)$ if $c_0 > 0$ for $t=0$ and $(c_{\frac{q^2-4q-2}{2}}, c_{\frac{q^2-2q}{2}}, c_{n-d}) = (\frac{q+1}{3}, 1, \frac{2q-4}{3})$ if $c_0 = 0$ for $t=0$; $(c_{\frac{q^2-4q-2}{2}}, c_{\frac{q^2-2q}{2}}, c_{n-d}) = (1,1,q-2)$ for $t=\frac{q}{2}-1$; $(c_{\frac{q^2-3q}{2}}, c_{n-d}) = (1,q-1)$ for $t=\frac{q}{2}$. Hence we get (LHS of (5)) $\le ((\binom{\gamma_2}{2}+\binom{\frac{3}{2}q}{2}+0\cdot(q-2))+((\binom{\frac{3}{2}q}{2}\cdot\frac{q+1}{3}+\binom{\frac{q}{2}-1}{2}+0\cdot\frac{2q-4}{3})\cdot(\tau_0-1)+((\binom{\frac{3}{2}q}{2}+\binom{\frac{q}{2}-1}{2}+0\cdot(q-2))\cdot\tau_{\frac{q}{2}-1}+((\binom{q-1}{2}+0\cdot(q-1))\cdot\tau_{\frac{q}{2}} = q^4 - \frac{3}{8}q^3 + q^2 + \frac{3}{2}q + 1 < \frac{q^5+q^4}{8} - \frac{3}{2}q^2 - 2q + 1$, a contradiction. One can get a contradiction for $q \equiv 1 \pmod 3$ similarly. Thus $a_0 = 0$.
Setting $i = n-d$, the maximum possible contributions of c_j's in (6) to the LHS of (5) are $(c_{\frac{q^2-5q}{2}}, c_{\frac{2q^2-7q}{4}}, c_{n-d}) = (\frac{q}{4}, 1, \frac{3}{4}q-1)$ for $t=0$; $(c_{\frac{q^2-5q}{2}}, c_{n-d}) = (1,q-1)$ for $t=\frac{q}{2}-1$; $(c_{\frac{q^2-3q}{2}}, c_{n-d}) = (1,q-1)$ for $t=\frac{q}{2}$. So, (LHS of (5)) $\le ((\binom{2q-1}{2}\frac{q}{4}+\binom{\frac{5}{4}q-1}{2}+0\cdot(\frac{3}{4}q-1))\cdot\tau_0+((\binom{2q-1}{2}+0\cdot(q-1))\cdot\tau_{\frac{q}{2}-1}+((\binom{q-1}{2}+0\cdot(q-1))\cdot\tau_{\frac{q}{2}} = q^4 + \frac{33}{32}q^3 - \frac{17}{16}q^2 - \frac{9}{4}q + 1 \le \frac{q^5+q^4}{8} - \frac{3}{2}q^2 - 2q + 1$, a contradiction. This completes the proof of (a).

Next, to prove (b) of Theorem 1, let $\mathcal{C}$ be a putative $[g_q(4,d),4,d]_q$ codes for $d = 2q^3-3q^2-2q+1$ for $q \ge 7$. We have $\gamma_0 = 2$, $\gamma_1 = 2q-1$ from (1). By Lemma 7, the spectrum of a γ_2-plane Δ is (A) $(\tau_{q-2}, \tau_{q-1}, \tau_{2q-2}, \tau_{2q-1}) = (1,2,q,q^2-2)$ or (B) $(\tau_{q-1}, \tau_{2q-2}, \tau_{2q-1}) = (3,q+1,q^2-3)$. By Lemma 5, an i-plane satisfies

$i \geq (q-2)q-(q+1) = q^2-3q-1$. Hence $a_i = 0$ for any $i < q^2-3q-1$. Assume that an i-plane contains a 2-point. Since $(\gamma_1-2)\theta_2+2 = n+2q$, we have $i \geq (\gamma_1-2)\theta_1+2-2q = (2q-3)\theta_1+2-2q = 2q^2-3q-1 > \theta_2$ for $q \geq 7$. Thus, $a_i = 0$ if $i < q^2-3q-1$ or $\theta_2 < i < 2q^2-3q-1$. Furthermore, using Lemmas 5 and 8, it can be proved that $a_i > 0$ implies $i \in \{q^2-3q-1, \cdots, q^2-2q-5, q^2-2q-1, \cdots, q^2-q-5, q^2-1, q^2+q-1, 2q^2-3q-1, \cdots, 2q^2-q-2\}$. From (2), we get

$$\sum_{i=q^2-3q-1}^{2q^2-q-4} \binom{2q^2-q-2-i}{2} a_i = q^2\lambda_2 + (-q^5+\frac{7}{2}q^4+2q^3-\frac{11}{2}q^2-2q+1). \quad (7)$$

For any i-plane through a t-line, Lemma 5(d) gives $\sum_j c_j = q$ and

$$\sum_j (2q^2-q-2-j)c_j = i+q+1-qt. \quad (8)$$

Suppose $a_i > 0$ for $i = q^2-3q-1+e$ with $0 \leq e \leq q-4$ and let δ be an i-plane. We may assume that Δ has spectrum (A). It follows from Lemma 8 and known results on $m_{q-2}(2,q)$ for small q that we have $i \leq q^2-2q-6$ for $q \geq 7$. So, we have $e \leq q-5$. Note that $a_i > 0$ implies $a_i = 1$ and $a_j = 0$ for $q^2-3q-1 \leq j \leq q^2-2q-6$, $j \neq i$. Now, we set $i = n-d$ in (8). If q is odd, the maximum possible contributions of c_j's in (8) to the LHS of (7) are $(c_{q^2-3q-1+e}, c_{n-d-e}, c_{n-d}) = (1,1,q-2)$ for $t = q-2$; $(c_{2q^2-3q-1}, c_{2q^2-\frac{3}{2}q-\frac{3}{2}}, c_{n-d}) = (\frac{q+1}{2}, 1, \frac{q-3}{2})$ for $t = q-1$; $(c_{2q^2-3q-1}, c_{n-d}) = (1, q-1)$ for $t = 2q-2$; $(c_{2q^2-2q-1}, c_{n-d}) = (1, q-1)$ for $t = 2q-1$. Hence we get (LHS of (7)) $\leq ((\binom{q^2+2q-1-e}{2} + \binom{e}{2})\tau_{q-2} + (\frac{q+1}{2}\binom{2q-1}{2} + \binom{\frac{q-1}{2}}{2})\tau_{q-1} + \binom{2q-1}{2}\tau_{2q-2} + \binom{q-1}{2}\tau_{2q-1} < ((\binom{q^2+2q-1}{2} + \binom{q-5}{2})\tau_{q-2} + (\frac{q+1}{2}\binom{2q-1}{2} + \binom{\frac{q-1}{2}}{2})\tau_{q-1} + \binom{2q-1}{2}\tau_{2q-2} + \binom{q-1}{2}\tau_{2q-1}$, giving $\lambda_2 < q^3 - \frac{5}{2}q^2 + \frac{5}{2}q + \frac{15}{4}$. On the other hand, we have $\lambda_2 = n - \theta_3 + \lambda_0 \geq (2q^3-q^2-3q-1) - (q^3+q^2+q+1) + ((q^2+q+1) - (q^2-2q-6)) = q^3-2q^2-q+5$, giving a contradiction for $q \geq 7$. One can also get a contradiction when q is even. Thus, $a_i = 0$ for $q^2-3q-1 \leq i \leq q^2-2q-5$. Similarly, we can get $a_i = 0$ for all $i < 2q^2-3q-1$ using Lemma 5 and (7). Finally, we investigate (7) and (8) with $i = n-d$ again. Assume q is odd. The maximum possible contributions of c_j's in (8) to the LHS of (7) on Δ are $(c_{2q^2-3q-1}, c_{2q^2-\frac{5}{2}q-\frac{3}{2}}, c_{n-d}) = (\frac{q+1}{2}, 1, \frac{q-3}{2})$ for $t = q-2$; $(c_{2q^2-3q-1}, c_{2q^2-\frac{3}{2}q-\frac{3}{2}}, c_{n-d}) = (\frac{q+1}{2}, 1, \frac{q-3}{2})$ for $t = q-1$; $(c_{2q^2-3q-1}, c_{n-d}) = (1, q-1)$ for $t = 2q-2$; $(c_{2q^2-q-3}, c_{n-d}) = (1, q-1)$ for $t = 2q-1$. Hence we get (LHS of (7)) $\leq (\frac{q+1}{2}\binom{2q-1}{2} + \binom{\frac{3q-1}{2}}{2})\tau_{q-2} + (\frac{q+1}{2}\binom{2q-1}{2} + \binom{\frac{q-1}{2}}{2})\tau_{q-1} + \binom{2q-1}{2}\tau_{2q-2} + \binom{q-1}{2}\tau_{2q-1}$, giving $\lambda_2 < q^3 - 3q^2 + \frac{3}{2}q + \frac{z}{8}$ where $z = 27$ (resp. 31) when Δ has spectrum (A) (resp. (B)). On the other hand, we have $\lambda_2 =$

$n - \theta_3 + \lambda_0 \geq (2q^3 - q^2 - 3q - 1) - (q^3 + q^2 + q + 1) = q^3 - 2q^2 - 4q - 2$, giving a contradiction for $q \geq 7$. One can get a contradiction similarly when q is even. This completes the proof of (b).

Finally, we give a sketch of the proof of (c) in Theorem 1. Let $\mathcal{C}$ be a putative $[g_q(4,d), 4, d]_q$ codes for $d = 2q^3 - rq^2 - q + 1$ for $3 \leq r \leq q - q/p$, $q = p^h$ with p prime. Let Δ be a γ_2-plane. Then we can prove the following.

Lemma 10 (Cf. [5])**.** (a) *On Δ, every line through a 2-point is a γ_1-line.*
(b) *Every i-line through a 2-point satisfies $\theta_2 < \gamma_2 - (q-1) \leq i \leq \gamma_2$.*
(c) *The multiset for Δ consists of two copies of the plane with an $(r,2)$-arc of lines deleted.*

Since the multiset given by two copies of Δ with the multiset for Δ deleted forms an $(r\theta_1, r; 2, q)$-minihyper and since $\gamma_0 = 2$, we can employ Lemma 9 to prove the part (c) of Lemma 10. Lemma 10 yields that Lemma 4.1 in [5] holds under our assumption "$5 \leq r \leq q - q/p$, $q = p^h$ with p prime" instead of "$q > 2(r-1), r \geq 5$". From a similar argument as in the proof for (a), we can show that $a_i > 0$ implies $q^2 - (r-1)q - (r-3) \leq i \leq q^2 - (r-2)q - (r-2)$ or $\gamma_2 - (q-1) \leq i \leq \gamma_2$. Then, one can get a contradiction as in the proof of Theorem 1.4 in [5]. This completes the proof of (c). □

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