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Five new (n, r)-arcs in PG(2, 29)[1]

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Abstract. An (n, r)-arc is a set of n points of a projective plane such that some r, but no $r+1$ of them, are collinear. The maximum size of an (n, r)-arc in PG(2, q) is denoted by $m_r(2, q)$. In this paper we continue our research, started in [8], and present five new (n, r)-arcs with parameters $(476, 18)$, $(500, 19)$, $(529, 20)$, $(564, 21)$ and $(592, 22)$. The constructed arcs improve the respective lower bounds on $m_r(2, q)$ in [8, 10].

1 Introduction

Let $\mathrm{GF}(q)$ denote the Galois field of q elements and $\mathrm{V}(3, q)$ be the vector space of row vectors of length three with entries in $\mathrm{GF}(q)$. Let $\mathrm{PG}(2, q)$ be the corresponding projective plane. The *points* (x_1, x_2, x_3) of $\mathrm{PG}(2, q)$ are the 1-dimensional subspaces of $\mathrm{V}(3, q)$. Subspaces of dimension two are called *lines*. The number of points and the number of lines in $\mathrm{PG}(2, q)$ is $q^2 + q + 1$. There are $q + 1$ points on every line and $q + 1$ lines through every point.

Definition 1. *An (n, r)-arc is a set of n points of a projective plane such that some r, but no $r + 1$ of them, are collinear.*

Definition 2. *An (l, t)-blocking set S in $\mathrm{PG}(2, q)$ is a set of l points such that every line of $\mathrm{PG}(2, q)$ intersects S in at least t points, and there is a line intersecting S in exactly t points.*

An (n, r)-arc is the complement of a $(q^2 + q + 1 - n, q + 1 - r)$-blocking set in a projective plane and conversely.

Definition 3. *Let M be a set of points in any plane. An i-secant is a line meeting M in exactly i points. Define τ_i as the number of i-secants to a set M.*

In terms of τ_i the definitions of an (n, r)-arc and an (l, t)-blocking set become the following: An (n, r)-arc is a set of n points of a projective

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plane for which $\tau_i \geq 0$ for $i < r$, $\tau_r > 0$ and $\tau_i = 0$ when $i > r$. An (l, t)-blocking set is a set of l points of a projective plane for which $\tau_i = 0$ for $i < t$, $\tau_t > 0$ and $\tau_i \geq 0$ when $i > t$.

A survey of (n, r)-arcs with the best known results was presented in [9]. After this publication many improvements were obtained in [4], [5] and [3]. Summarizing these improvements, Ball and Hirschfeld [2] presented a new table with bounds on $m_r(2, q)$ for $q \leq 19$. It follows from these tables that the exact values of $m_r(2, q)$ are known only for $q \leq 9$. A survey of the new improvements in recent years can be found in the online table for $m_r(2, q)$, $q \leq 19$, maintained by S. Ball [1] or in [8]. New results and tables with lower and upper bounds on $m_r(2, q)$ for $q = 23$, and $q = 25, 27$ are presented in [6] and [7] respectively.

A short description of our search method can be found in [6–8]. In order to present the results in more concise form, the points in PG(2,29) are in lexicographic order and each point is associated with its number. For example: the first point is $(0, 0, 1)$, the point $(0, 1, 1)$ has number 3, the point $(1, 9, 11)$ has number 303, the point $(1, 19, 27)$ has number 609.

The following lower bounds on $m_r(2, 29)$ are given in [10] and [8].

The lower bounds on $m_r(2, 29)$ in [10] and [8]

r	2	3	4	5	6	7	8	9	10
$m_r(2, 29)$	**30**	43	70	94	126	146	181	201	233
r	11	12	13	14	15	16	17	18	19
$m_r(2, 29)$	258	291	325	361	**407**	**436**	452	474	499
r	20	21	22	23	24	25	26	27	28
$m_r(2, 29)$	521	563	580	619	658	683	715	755	784

The arcs marked in bold are optimal. In this paper we improve the lower bounds for $r = 18, 19, 20, 21, 22$ by constructing five new (n, r)-arcs.

2 The new arcs in PG(2, 29)

Theorem 1. *There exist a $(476, 18)$-arc, a $(500, 19)$-arc, a $(529, 20)$-arc, a $(564, 21)$-arc and a $(592, 22)$-arc in* PG(2,29)*.*

Proof. 1. The set of points having numbers 3 7 12 14 15 19 20 21 22 27 28 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 51 52 53 54 55 56 57 58 59 62 68 69 71 73 74 75 76 78 80 81 87 91 96 98 100 101 104 106 107 109 111 112 116 119 120 123 125 126 128 133 136 137 139 140 142 145 147 148 150

151 154 158 161 163 165 169 172 173 175 176 178 181 183 185 188 193 196 198 200 203 204 205 206 209 211 218 219 220 221 222 228 230 233 234 237 238 240 242 243 244 246 253 254 255 259 264 267 268 270 274 275 277 278 280 281 285 287 288 291 292 294 297 298 302 305 308 311 315 316 317 319 326 328 329 330 333 334 337 338 340 341 342 343 345 353 355 359 360 361 363 364 365 366 368 369 370 374 376 380 381 383 388 389 392 393 394 395 398 399 404 406 407 408 411 413 414 415 417 422 423 428 430 432 434 440 441 444 445 449 450 451 453 454 458 459 462 463 467 469 472 473 475 476 477 484 485 486 492 494 495 497 498 501 503 509 510 516 517 518 521 522 528 529 530 532 535 536 541 542 543 545 547 548 549 555 557 560 561 564 566 569 574 575 576 578 580 583 584 585 586 590 591 593 600 602 603 607 609 610 611 613 615 621 622 625 626 629 630 635 636 638 641 642 643 646 650 652 653 656 657 663 666 667 668 669 670 673 676 677 678 679 688 689 691 694 697 699 704 708 709 710 712 713 715 717 721 723 726 727 728 729 732 735 739 744 748 751 753 754 755 756 759 763 766 767 768 769 773 774 775 778 782 787 788 789 790 791 794 797 802 805 808 809 810 811 812 815 817 819 820 822 827 828 829 830 835 836 837 838 839 840 842 843 848 849 851 852 856 859 861 863 864 866 867 forms a (395,12)-blocking set in PG(2,29) with secant distribution

$$\tau_{12} = 272,\ \tau_{13} = 275,\ \tau_{14} = 203,\ \tau_{15} = 77,\ \tau_{16} = 16,$$

$$\tau_{18} = 1,\ \tau_{26} = 6,\ \tau_{27} = 6,\ \tau_{28} = 13,\ \tau_{29} = 2$$

The complement of this blocking set is a $(476, 18)$-arc in PG(2, 29).

2. The set of points having numbers 1 2 4 8 10 15 16 17 18 20 23 25 29 31 33 37 39 44 45 46 47 52 54 58 65 66 67 71 74 75 78 82 83 88 90 91 94 96 98 101 106 109 111 113 117 123 125 126 127 128 130 135 137 138 139 140 142 143 151 152 153 155 157 158 159 164 165 166 168 170 171 172 175 179 180 181 182 185 189 196 199 200 201 202 206 209 210 212 216 223 227 229 230 233 235 236 237 240 244 246 247 250 251 253 257 260 261 262 263 266 267 271 272 273 275 280 282 283 284 288 289 292 293 294 295 303 310 316 318 319 320 324 325 328 329 333 334 337 338 342 343 346 347 350 351 354 356 359 362 363 364 365 366 367 373 378 381 384 385 387 388 392 393 394 395 397 399 400 402 403 406 408 410 411 415 418 421 424 427 430 434 435 438 443 447 448 449 451 452 454 455 456 460 463 464 465 467 472 476 477 478 480 481 483 484 485 489 494 495 498 502 505 508 511 514 517 521 522 526 529 530 532 533 535 536 538 539 540 542 544 545 547 548 551 553 554 557 559 562 565 566 567 568 569 570 576 578 585 586 589 590 594 595 598 599 603 604 607 608 611 612 613 614 616 629 635 637 638 639 640 641 643 644 648 649 650 652 654 655 657 659 660 661 665 666 670 671 672 675 679 681 682 685 686 688 691 692 695 696 697 699 702 703 705 709 716 720 722 723 726 730 731 732 733 736 747 750 751 752 753 759 760 761 762 763 764 766 767 768 771 773 774 775 777 779 780 781 790 792 793 794 795 797 802 804 805 806 807 809 815 816 819 821 823 826 831 834 836 838 841 842 843 849

850 854 857 858 861 865 866 867 forms a (371,11)-blocking set in PG(2,29) with secant distribution

$$\tau_{11} = 247,\ \tau_{12} = 261,\ \tau_{13} = 178,\ \tau_{14} = 93,\ \tau_{15} = 39,\ \tau_{16} = 22,$$

$$\tau_{17} = 11,\ \tau_{18} = 3,\ \tau_{19} = 1,\ \tau_{28} = 4,\ \tau_{29} = 4,\ \ \tau_{30} = 8$$

The complement of this blocking set is a $(500, 19)$-arc in PG(2, 29).

3. The set of points having numbers 1 2 4 10 15 16 17 18 23 25 31 33 39 44 45 46 47 52 58 66 67 71 74 75 78 82 83 84 90 91 94 96 98 101 106 109 111 113 116 117 123 126 127 128 130 131 135 137 138 139 140 142 151 153 155 157 158 159 164 165 166 168 170 171 172 179 180 181 185 196 199 200 201 202 209 210 212 216 223 227 229 230 233 235 236 237 240 244 246 247 250 251 253 257 260 261 263 266 267 271 272 273 275 277 278 280 282 283 284 288 292 293 294 295 297 303 310 318 320 324 325 328 329 333 334 335 337 338 342 343 347 350 351 354 356 359 362 363 364 365 366 367 372 375 378 381 384 385 387 388 390 392 393 394 395 399 402 403 406 408 410 411 415 418 421 424 430 434 435 438 443 447 448 449 451 452 455 456 460 465 467 472 476 477 480 481 483 484 485 489 494 495 497 498 502 508 511 514 517 521 522 526 529 530 533 537 538 539 540 542 544 545 547 548 551 553 554 557 560 565 566 567 568 569 570 573 576 578 581 585 589 590 594 595 597 598 599 603 604 607 608 611 612 614 622 629 635 637 638 639 640 644 648 649 650 652 654 655 657 659 660 661 665 666 671 672 675 679 681 682 685 686 688 692 695 696 697 699 702 703 705 709 716 720 722 723 726 730 731 732 733 736 747 751 752 753 760 761 762 764 766 767 768 773 774 775 777 779 781 790 792 793 794 795 797 801 802 804 805 806 809 815 816 819 821 823 826 831 834 836 838 841 842 848 849 850 854 857 858 861 865 866 forms a (342,10)-blocking set in PG(2,29) with secant distribution

$$\tau_{10} = 202,\ \tau_{11} = 289,\ \tau_{12} = 202,\ \tau_{13} = 109,\ \tau_{14} = 30,$$

$$\tau_{15} = 18,\ \tau_{16} = 7,\ \tau_{29} = 2,\ \tau_{30} = 12$$

The complement of this blocking set is a $(529, 20)$-arc in PG(2, 29).

4. The set of points having numbers 1 2 3 4 10 11 22 23 29 30 39 42 44 45 46 47 49 58 59 61 62 66 67 69 71 74 78 80 82 83 87 88 89 90 91 96 98 109 111 116 117 125 127 128 131 134 137 138 140 151 152 155 157 158 159 164 165 171 172 178 179 180 181 187 200 201 202 203 205 206 207 209 210 212 219 220 227 229 230 232 233 235 240 246 247 250 251 257 262 266 267 271 272 283 284 288 289 292 293 297 300 304 309 316 319 320 328 329 335 336 342 343 346 347 351 356 359 363 364 365 366 370 373 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 410 411 418 420 421 424 425 427 434 435 438 445 447 448 451 452 454 455 456 458 465 467 474 476 477 478 483 484 485 487 494 497 498 505 507 508 511 512

514 521 522 527 529 530 532 534 543 545 547 548 550 554 559 566 567 568 569 576 581 586 589 590 596 597 603 604 607 611 612 613 616 623 628 629 635 638 639 640 643 644 648 649 658 660 661 665 666 671 675 681 682 685 686 692 696 702 703 705 712 713 720 722 723 726 729 730 731 732 733 750 751 752 753 754 756 757 760 764 767 768 773 774 777 781 786 793 794 795 798 801 804 805 814 815 816 821 826 831 836 841 842 845 846 849 850 852 854 857 858 861 863 865 866 870 forms a (307,9)-blocking set in PG(2,29) with secant distribution

$$\tau_9 = 272,\ \tau_{10} = 269,\ \tau_{11} = 192,\ \tau_{12} = 65, \tau_{13} = 35,$$

$$\tau_{14} = 18,\ \tau_{15} = 4,\ \tau_{16} = 4,\ \tau_{19} = 1,\ \tau_{30} = 11$$

The complement of this blocking set is a $(564, 21)$-arc in PG(2, 29).

5. The set of points having numbers 1 2 4 16 17 20 21 29 39 43 44 46 47 48 52 66 67 71 74 75 78 82 83 92 94 96 98 101 106 109 111 113 121 123 125 126 128 131 137 139 140 142 152 153 155 157 166 168 170 171 179 180 182 185 196 199 200 201 202 206 209 210 211 213 216 219 220 223 229 230 233 235 236 237 240 244 253 260 261 262 263 266 267 275 280 285 288 289 293 294 297 303 310 316 319 324 325 328 334 337 343 346 347 350 351 356 359 362 367 370 373 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 418 421 424 427 432 435 438 439 443 449 451 452 454 465 467 478 480 481 483 489 494 495 501 502 505 508 511 514 526 532 535 538 539 542 545 551 554 559 562 565 570 573 576 578 581 582 585 586 589 595 598 599 604 607 608 610 611 613 616 617 622 629 635 638 640 643 644 646 648 652 657 665 666 670 671 672 679 681 685 688 693 695 696 697 699 702 703 704 705 709 711 716 722 723 726 730 731 733 736 747 750 751 752 753 761 762 764 766 775 777 779 780 790 792 793 794 795 804 806 807 809 821 823 826 831 834 836 838 849 850 851 854 857 858 861 865 866 forms a (279,8)-blocking set in PG(2,29) with secant distribution

$$\tau_8 = 256,\ \tau_9 = 287,\ \tau_{10} = 170,\ \tau_{11} = 70,\ \tau_{12} = 51, \tau_{13} = 16,$$

$$\tau_{14} = 6,\ \tau_{15} = 2,\ \tau_{16} = 3,\ \tau_{17} = 1,\ \tau_{30} = 9$$

The complement of this blocking set is a $(592, 22)$-arc in PG(2, 29).

□

The new lower bounds on $m_r(2,29)$

r	2	3	4	5	6	7	8	9	10
$m_r(2,29)$	30	43	70	94	126	146	181	201	233
r	11	12	13	14	15	16	17	18	19
$m_r(2,29)$	258	291	325	361	407	436	452	***476***	***500***
r	20	21	22	23	24	25	26	27	28
$m_r(2,29)$	***529***	***564***	***592***	619	658	683	715	755	784

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