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Enumeration of some optimal ternary constant-weight codes

Galina Bogdanova, Todor Todorov[1] galina, todor@math.bas.bg
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
P.O. Box 323, 5000 Veliko Tarnovo, BULGARIA

Abstract. We consider the problem of classification of optimal ternary constant-weight codes. We use combinatorial and computer methods to find inequivalent codes for some cases for $3 \leq d \leq n \leq 9$.

1 Introduction

A ternary (n, M, d) code consists of M vectors (called codewords) of length n over the alphabet {0,1,2}, such that any two codewords differ in at least d positions.

A code is called *constant weight* if all the codewords have the same Hamming weight. Constant weight codes have been studied by many authors [10],[11],[7],[2],[1].

We will use the following notation for the parameters of a ternary constant-weight (TCW)code: (n, M, d, w). Let $A_3(n, d, w)$ denote the largest possible value M, for which there exists an (n, M, d, w) code. TCW codes of size $M = A_3(n, d, w)$ are called *optimal*.

Initially, bounds and exact values of the function $A_3(n, d, w)$ were presented in [7] and the recent results may be found in [8]. In this paper we explore the problem of enumerating (up to equivalence) optimal TCW codes with $3 \leq d \leq n \leq 9$.

Combinatorial and computer methods can be used to classify optimal codes. Enumeration of TCW codes by computer methods is presented in Section 2. The results which have been obtained are presented in Section 3.

2 Enumeration of TCW codes by computer methods

Definition 1 *Two ternary constant-weight codes are equivalent if one of them can be obtained from the other by transformations of the following types:*
- *permutation of the coordinates of the code;*
- *permutation of the alphabet symbols appearing in a fixed position.*

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Bounds and exact values for the size of the codes could be found in [8, 5].

A known upper bound $A_3(n, d, w) \leq M$ may be improved after an exhaustive computer search for a code with these parameters and size M. This search can in fact be conveniently described as a search for a clique of size M in the following graph. Consider the graph where the vertex set corresponds to the words of length n and Hamming weight w and two vertices are joined by an edge if the Hamming distance between the corresponding words is greater than or equal to d. With a maximum clique algorithm, we would find the exact value of $A_3(n, d, w)$ but this direct approach is computationally feasible only for very small parameters. We may then perhaps relax the goal and just try to lower the upper bound. In any case, to speed up the search, it is essential to handle the large automorphism group of the constructed graph. This may be done in the following way by utilizing the Johnson-type bounds and removing equivalent copies of partial codes. We know that an (n, M, d, w) code can be shortened to get $(n-1, M', d, w)$ and $(n-1, M'', d, w-1)$ subcodes, where

$$M' \geq \frac{n-w}{n}.M, M'' \geq \frac{w}{n(q-1)}.M$$

Therefore, we may construct a code C by classifying all such subcodes (for one of these two alternatives), and then use the clique-finding approach to find the rest of the words in C.

The method we use is described in [7], [8], [3] and [9].

The two basic steps are:

- Finding all inequivalent possibilities for subcode C';
- Extending any of them to the size of C.

For the application of this method it is crucially important to have an effective algorithm for determining code equivalence.

We implement the steps 1 and 2 using our own, specifically developed, computer algorithms and programs. These algorithms are implemented in the computer package QPlus [4]. Some of the results are also verified using Q-Extension software [6].

3 Results

Let $\#(n, M, d, w)$ denote the number of inequivalent TCW codes with the specified parameters. The computer results are described by the following Theorem:

Theorem 1 *#(3,3,3,2)=1, #(4,4,3,2)=1, #(4,2,4,2)=1, #(4,8,3,3)=1, #(4,2,4,3)=1, #(5,5,3,2)=1, #(5,2,4,2)=1, #(5,12,3,3)=1, #(5,5,4,3)=1, #(5,2,5,3)=1, #(5,10,3,4)=64, #(5,5,4,4)=1, #(5,2,5,4)=1, #(6,6,3,2)=2, #(6,3,4,2)=1, #(6,18,3,3)=54, #(6,8,4,3)=3, #(6,4,5,3)=1, #(6,2,6,3)=1, #(6,15,4,4)=1, #(6,4,5,4)=1, #(6,3,6,4)=1,* $\#(6, 24, 3, 5) \geq 20$*, #(6,12,4,5)=1,*

#(6,3,5,5)=1, #(6,2,6,5)=1, #(7,7,3,2)=2, #(7,3,4,2)=1, #(7,14,4,3)=1, #(7,4,5,3)=2, #(7,2,6,3)=1, #(7,7,5,4)=45, #(7,3,6,4)=3, #(7,2,7,4)=1, #(7,3,6,5)=4, #(7,2,7,5)=1, #(7,9,5,5)=2, $\#(7,14,4,6) \geq 74$*, #(7,7,5,6)=1, #(7,2,6,6)=1, #(7,2,7,6)=1, #(8,8,3,2)=3, #(8,4,4,2)=1, #(8,5,5,3)=1, #(8,2,6,3)=1, #(8,5,6,4)=2, #(8,2,7,4)=2, #(8,2,8,4)=1, #(8,8,6,5)=5, #(8,3,7,5)=3, #(8,2,8,5)=1, #(8,8,6,6)=22, #(8,3,7,6)=2, #(8,2,8,6)=1, #(8,16,5,7)=1, #(8,4,6,7)=2, #(8,2,7,7)=2, #(8,2,8,7)=1, #(9,9,3,2)=4, #(9,4,4,2)=1, #(9,6,5,3)=2, #(9,3,6,3)=1, #(9,3,7,4)=1, #(9,2,8,4)=1, #(9,5,7,5)=1,#(9,3,8,5)=1, #(9,2,9,5)=1, #(9,6,7,6)=12, #(9,3,8,6)=4, #(9,3,9,6)=1, #(9,5,7,7)=11, #(9,3,8,7)=1, #(9,2,9,7)=1, #(9,3,7,8)=1, #(9,2,8,8)=2, #(9,2,9,8)=1.*

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